Fifth sheet of exercises

1. Pure endowment life insurance

A woman signed a pure endowment life insurance contract in the first of January 2018 (*i.e.* $A(t) = 0$ for $t < \tau$ and $A(\tau) > 0$). The contract is valid until the first of January 2026, thus $\tau = 8$. We suppose that the future lifetime of the policyholder T follows the uniform law $\mathcal{U}([0, 10])$, the capital function is given by $K(t) = (1 + 4\%)^{\lfloor t \rfloor}$. We suppose that the policyholder has to pay an advanced premium of 600 euro in the first of January of each year, starting from the date of signing the contract. So, we deduce that the premium function is given by

$$\Pi(t) = \sum_{i=0}^{7} 600\mathbf{1}_{[i,\infty)}(t), \ \ t \in \mathbb{R}_+.$$

Calculate $A(8)$ such that Π is a net premium function.

2. Accumulated force of mortality for Y

We consider a life insurance police (LIP) where the future lifetime of the policyholder $T \sim \mathcal{U}([0, 8])$.

(a) Determine the maximal future lifetime $t_{\max}$ (see Definition 1.2.1. in the course and exercise 2 in the second sheet of exercises).

(b) We suppose that the terminal time of the police is $\tau = 7$. Calculate $\triangle\Lambda_Y(t)$, for $t \in (0, 7]$, where Λ_Y is the accumulated force of mortality for Y and $\triangle\Lambda_Y(t) := \Lambda_Y(t) - \Lambda_Y(t-)$.

(c) We recall that $\tau \in (0, t_{\max}]$. We suppose that $\tau = t_{\max}$, calculate $\triangle\Lambda_Y(t)$, for $t \in (0, \tau]$.

3. Loss of the insurance company

We consider a LIP such that : the terminal time of the police is $\tau = +\infty$, the future lifetime of the policyholder T follows the exponential law $\mathcal{E}(1/9)$, the payment spectrum is given by $A(u) = u$ for $u \in \mathbb{R}_+$, the capital function is given by $K(u) = e^{\frac{3u}{100}}$ for $u \in \mathbb{R}_+$, and the premium function is given by $\Pi(u) = \frac{100u}{127}$, for $u \in \mathbb{R}_+$.

(a) Prove that Π is a net premium function.

(b) Calculate the prospective net premium reserve $V(t)$, for $t \geq 0$.

(c) Calculate the variance of the loss of the insurance company until time $t \in \mathbb{R}_+$ (*i.e.* $\mathrm{Var}(L(t))$).

(d) Calculate the variance of the present value (*i.e.* $\mathrm{Var}(B)$).

4. Unimodal function

We recall that a function $f : (0, \infty) \to \mathbb{R}_+$ is called unimodal with mode at $x \in (0, \infty)$ if f is strictly increasing on $(0, x)$, and strictly decreasing on (x, ∞). We recall that the Gamma distribution $\Gamma(\alpha, \beta)$ for $\alpha, \beta > 0$ has the density

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \;\; x > 0,$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Prove that for $\alpha > 1$ the density f is unimodal with mode at $\frac{\alpha-1}{\beta}$.