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Fourth sheet of exercises : Hattendorf's theorem

1. Canonical filtration

For $t \in \mathbb{R}_+$, $N_t := \mathbf{1}_{\{Y \le t\}}$ where $Y := \min(T, \tau)$ is the random time of benefit. The canonical filtration $(\mathscr{F}_t)_{t\in\mathbb{R}_+}$ is defined as $\mathscr{F}_t := \sigma(N_s : s \in [0,t]) = \sigma(\{Y \le s\} : s \in [0,t])$, $t \in \mathbb{R}_+$. Prove that

$$\mathscr{F}_t = \sigma(\min\{Y,t\}, \mathbf{1}_{\{Y=t\}}),\ t \in \mathbb{R}_+.$$

2. Accumulated force of mortality

We consider Λ_Y the accumulated force of mortality for Y. We consider also the following two statements :

(i) We have $\lim_{t\uparrow\tau} \Lambda_Y(t) = \infty$.

(ii) We have either $\tau = \infty$ or both $\tau < \infty$ and $F_Y(\tau-) = 1$.

Prove that

(a) $(i) \Longrightarrow (ii)$,

(b) If Y is has a continuous density f_Y, then $(ii) \Longrightarrow (i)$.

3. Martingale

We assume that $\tau = \infty$ and $T \sim \mathcal{U}([0,1])$, where $\mathcal{U}([0,1])$ denotes the uniform law on $[0,1]$.

(a) Determine the martingale process M defined as $M_t = N_t - \int_{[0,t\wedge Y]} d\Lambda_Y(u)$, $t \in \mathbb{R}_+$.

(b) Verify that $E[M_t] = 0, \forall t \in \mathbb{R}_+$.

4. Loss of the insurance company

For $t \in \mathbb{R}_+$, $L(t)$ denotes the loss of the insurance company until time $t \in \mathbb{R}_+$, defined as $L(t) = E[B|\mathscr{F}_t]$. Prove that

$$L(t) = \left(\frac{A(Y)}{K(Y)} - \int_{[0,Y[} \frac{1}{K(s)} d\Pi(s)\right)\mathbf{1}_{\{Y\le t\}} + \left(\frac{V(t)}{K(t)} - \int_{[0,t)} \frac{1}{K(s)} d\Pi(s)\right)\mathbf{1}_{\{Y>t\}}.$$