SPDEs 2016/17 Exercise Sheet 8

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Mild solutions and related solution concepts

We consider the stochastic evolution equation

$$dX_t = (AX_t + F_t(X_t))dt + B_t(X_t)dW_t, \tag{1}$$

where W is cylindrical Brownian motion on a separable Hilbert space U, A is the generator of a strongly continuous semigroup on a separable Hilbert space H, and F and B are nonlinear mappings on appropriate spaces.

a) Assuming that F and B vanish, describe the relation between mild solutions of (1) and solutions of the abstract Cauchy problem $dX_t/dt = AX_t$ [EN99, Section II.6].

b) Assuming that B vanishes, F depends only on time, and H is finite-dimensional, show that the definition of mild solutions of (1) coincides with the variation of constants formula for ordinary differential equations [Arn92, Section 29].

c) Assuming that F vanishes and B is constant, show that the notion of mild solutions of (1) corresponds to the integral representation of Ornstein-Uhlenbeck processes (see [Jac96] for definitions and a historical context).

References

[Arn92] Vladimir I. Arnold. *Ordinary Differential Equations*. Universitext. Berlin Heidelberg: Springer, 1992.

[EN99] Klaus-Jochen Engel and Rainer Nagel. *One-parameter semigroups for linear evolution equations*. Vol. 194. Springer Science, 1999.

[Jac96] Martin Jacobsen. “Laplace and the origin of the Ornstein-Uhlenbeck process”. In: *Bernoulli* 2.3 (1996), pp. 271–286.