SPDEs 2016/17 Exercise Sheet 5

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Due date: Thursday November 24, 2016

Setting

Let U be a separable Hilbert space.

5.1. Existence of cylindrical Brownian motion

Show that there exists a Q-cylindrical Brownian motion for any symmetric non-negative $Q \in L(U)$.

5.2. Cylindrical Brownian motion is Brownian motion on a larger Hilbert space

Show that the following statement holds: if W is cylindrical Q-Brownian motion with $Q \in L(U)$ and $i : U \to \tilde{U}$ is a Hilbert-Schmidt embedding into another Hilbert space $\tilde{U}$, then $\tilde{W}$ is cylindrical $\tilde{Q}$-Brownian motion with $\tilde{Q} \in L_1(\tilde{U})$, where $\tilde{W} := W \circ (I_{L^2(\mathbb{R}_+)} \otimes i^*) : L^2(\mathbb{R}_+;\tilde{U}) \to L^2(\Omega)$ and $\tilde{Q} := i \circ Q \circ i^*$.

Note: we have seen in the lecture that cylindrical Brownian motion with nuclear covariance operator can be identified with Brownian motion.

5.3. Existence of Hilbert-Schmidt embeddings

Show that for any separable Hilbert space U there is a Hilbert-Schmidt embedding $i : U \to \tilde{U}$ into a separable Hilbert space $\tilde{U}$.

5.4. Existence of cylindrical Brownian motion with respect to a filtration

Let W be Q-cylindrical Brownian motion for some $Q \in L(U)$. Show that there exists a right-continuous and complete filtration $(\mathscr{F}_t)$ such that W is Q-cylindrical Brownian motion with respect to $(\mathscr{F}_t)$.