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Discrete Time Finance
Sixth exercise

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1. **Replication**

   Let $\Omega = \{\omega_1, \dots, \omega_4\}$, $\mathcal{F} = 2^\Omega$ and let $\mathbb{P}$ a strictly positive probability measure on $(\Omega, \mathcal{F})$. Set $S_t^0 := (1+r)^t$ for $r = \frac{1}{4}$ and $t \in \{0, \dots, 2\}$ and suppose the risky asset S^1 has the evolution

$$
\begin{tikzcd}
 & & 12 = S_2^1(\omega_1) \\
 & 8 \arrow[ur] \arrow[r] & 5 = S_2^1(\omega_2) \\
4 \arrow[ur] \arrow[dr] & & 3 = S_2^1(\omega_3) \\
 & 2 \arrow[ur] \arrow[r] & 1 = S_2^1(\omega_4)
\end{tikzcd}
$$

   (a) Consider the option $C = \max_{t=0,1,2} S_t^1 - S_2^1$. (3)
   Compute the arbitrage-free price of the discounted claim $H := C/S_2^0$.

   (b) Compute a replicating strategy for H. (3)

   Points for Question 1: 6

2. **The Space L^0**

   Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Define the mapping $d : L^0 \times L^0 \to \mathbb{R}_+$ by $d(X, Y) := \mathbb{E}[|X - Y| \wedge 1]$.
   Show that:

   (a) d defines a metric. (2)

   (b) For a sequence $(X_n) \subseteq L^0$ and $X \in L^0$ the following are equivalent: (2)
   - $d(X_n, X) \to 0$.
   - (X_n) converges to X in probability.

   (c) (L^0, d) is a topological vector space. (2)

   Points for Question 2: 6

You can achieve a total of **12** points for this sheet.