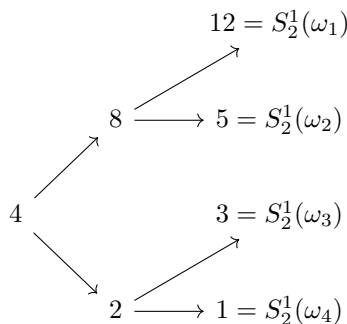


1. Replication

Let $\Omega = \{\omega_1, \dots, \omega_4\}$, $\mathcal{F} = 2^\Omega$ and let \mathbb{P} a strictly positive probability measure on (Ω, \mathcal{F}) . Set $S_t^0 := (1+r)^t$ for $r = \frac{1}{4}$ and $t \in \{0, \dots, 2\}$ and suppose the risky asset S^1 has the evolution



(a) Consider the option $C = \max_{t=0,1,2} S_t^1 - S_2^1$. (3)

Compute the arbitrage-free price of the discounted claim $H := C/S_2^0$.

(b) Compute a replicating strategy for H . (3)

Points for Question 1: 6

2. The Space L^0

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Define the mapping $d : L^0 \times L^0 \rightarrow \mathbb{R}_+$ by $d(X, Y) := \mathbb{E}[|X - Y| \wedge 1]$. Show that:

(a) d defines a metric. (2)

(b) For a sequence $(X_n) \subseteq L^0$ and $X \in L^0$ the following are equivalent: (2)

- $d(X_n, X) \rightarrow 0$.
- (X_n) converges to X in probability.

(c) (L^0, d) is a topological vector space. (2)

Points for Question 2: 6

You can achieve a total of **12** points for this sheet.