Stochastische Prozesse

WS 15/16

Vorlesung: Prof. Dr. Thorsten Schmidt
Exercise: Dr. Tolulope Fadina
http://www.stochastik.uni-freiburg.de/lehre/2015WiSe/inhalte/2015WiSeStochProz

Exercise 3

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Problem 1 (4 Points). (*a*) We say that a random variable η has the exponential distribution of rate $\lambda > 0$ if:

$$\mathbb{P}\{\eta > t\} = e^{-\lambda t} \quad \text{for all } t \geq 0.$$

Show that a random variable η with exponential distribution satisfies

$$\mathbb{P}\{\eta > t + s\} = \mathbb{P}\{\eta > t\}\mathbb{P}\{\eta > s\}.$$

Hint: When the probabilities are replaced by exponents, the equality should become obvious.

(*b*) Let $X = (X_t)_{t\geq 0}$ be a Poisson process with parameter $\lambda > 0$.
Show that

$$Y_t = 2^{X_t} \exp(-\lambda t)$$

is a martingale with respect to the filtration $(\mathcal{F}_t)_{t\geq 0}$ generated by the family of random variables $\{X_s : s \in [0, t]\}$.

Problem 2 (4 Points). Let $X = (X_t)_{t\geq 0}$ be a Brownian motion.

(*a*) Show that

$$|X_t|^2 - t$$

is a martingale.
Hint: Use the fact that $X_t - X_s$ is independent of $\mathcal{F}_s$ if $s < t$.

(*b*) Show that

$$\exp(X_t) \exp(-t/2)$$

is a martingale.

Problem 3 (4 Points). Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space with random variables X and Y such that X and Y are independents. Prove that

$$E[g(X, Y)|Y = y] = E[g(X, y)]$$

where g is a bounded measurable function.
Hint: Use the Monotone Class Theorem.