UNI FREIBURG

Stochastische Prozesse **WS 15/16**

Vorlesung: Prof. Dr. Thorsten Schmidt
Exercise: Dr. Tolulope Fadina
http://www.stochastik.uni-freiburg.de/lehre/2015WiSe/inhalte/2015WiSeStochProz

Exercise 13

Submission: 09-02-2016

Problem 1 (4 Points). Let B be a Brownian motion and define the $\mathbb{R}^2_+$-valued process X by $X_i(t) = (\sqrt{x_i} + B(t))^2$ for $i = 1, 2$, and for some $x \in \mathbb{R}^2$ such that X satisfies

$$\begin{aligned} dX_1 &= dt + 2\sqrt{X_1}dW, \\ dX_2 &= dt + 2\sqrt{X_2}dW, \\ X(0) &= x \end{aligned}$$

Is X an affine process?

Problem 2 (4 Points). Compute the characteristic function of $X(t)$ and verify your result concerning the (supposed) affine property of X.

Problem 3 (4 Points). Let $b, \sigma > 0$ and $\beta \in \mathbb{R}$, and consider the affine process

$$dX = (b + \beta X)dt + \sigma\sqrt{X}dW, \quad X(0) = x \in \mathbb{R}_+,$$

with state space $\mathbb{R}_+$.
Compute the corresponding system of Riccati equations.

Problem 4 (4 Points). Consider the Riccati differential equation

$$\partial_t G = aG^2 + bG - c, \quad G(0, u) = u$$

where $a, b, c \in \mathbb{C}$ and $u \in \mathbb{C}$, with $a \neq 0$ and $b^2 + 4ac \in \mathbb{C} \setminus \mathbb{R}$. Let $\sqrt{\cdot}$ denote the analytic extension of the real square root to $\mathbb{C} \setminus \mathbb{R}_-$, and define $\theta = \sqrt{b^2 + 4ac}$.
Show that the function

$$G(t, u) = \frac{2c(e^{\theta t} - 1) - (\theta(e^{\theta t} + 1) + b(e^{\theta t} - 1))u}{\theta(e^{\theta t} + 1) - b(e^{\theta t} - 1) - 2a(e^{\theta t} - 1)u}$$

is the unique solution of the Riccati differential equation on its maximum interval of existence $[0, t_+(u))$.