Unbounded Liabilities, Reserve Capital Requirements and the Taxpayer Put Option *

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Abstract

When firms access unbounded liability exposures and are granted limited liability, then an all equity firm holds a call option, whereby it receives a free option to put losses back to the taxpayers. We call this option the taxpayer put, where the strike is the negative of the level of reserve capital at stake in the firm. We contribute by (i) valuing this taxpayer put, and (ii) determining the level for reserve capital without a reference to ratings. Reserve capital levels are designed to mitigate the adverse incentives for unnecessary risk introduced by the taxpayer put at the firm level. In our approach, the level of reserve capital is set to make the aggregate risk of the firm externally acceptable, where the specific form of acceptability employed is positive expectation under a concave distortion of the cash flow distribution. It is observed that in the presence of the taxpayer put, debt holders may not be relied upon to monitor risk as their interests are partially aligned with equity holders by participating in the taxpayer put. Furthermore, the taxpayer put leads to an equity pricing model associated with a market discipline that punishes perceived cash shortfalls.

1 Introduction

Large players in the financial markets can place the financial system, and the real economy, at risk when they are insufficiently capitalized. Such observations have led to renewed calls for the regulation of large market participants. In fact, one of the stated objectives in the 2009 London meeting of the G20 in its communiqué is "to extend regulation and oversight to all systemically important financial institutions, instruments and markets. This will include, for the first time, systemically important hedge funds." An analysis of the implications of positions taken by large complex financial institutions (LCFI's), like hedge funds, can also be of potential interest for other corporate entities, given that they may enter financial markets and position themselves in comparable ways. The novelty of the analysis we present are the implications of exposure to unlimited liabilities for financial analysis.

By contrast, in the classical Merton (1973, 1974, 1977) model of the firm we have random assets and fixed liabilities with the consequence that in the worst case when asset values drop to zero wiping out both equity and debt, there are no negative consequences for the rest of the economy. In our model, on the other hand, firms have exposure to unbounded and random liabilities making it possible that liabilities overshoot assets to such an extent that even after writing down both debt and equity to zero, there remains a substantial bill yet to be paid. The exercise of limited liability in such a circumstance results in a transfer of wealth from creditor counterparties to equity holders.

In the context of our model there are two important and distinct limited liabilities to be considered. The first is the classic limited liability of equity, that in the presence of debt allows equity holders to put losses in asset values back to debt holders. Additionally we have the limited liability of the firm itself. This allows the firm to put losses back to the general economy when liabilities are sufficiently excessive. Our focus in this paper will be on this second put option, that has heretofore been occasionally recognized in financial analysis (see for example, Hovakimian and Kane (2000), Kane (1989) and John, Nair and Senbet (2009)). Our contribution is to both recognize its presence and provide a method to estimate its value.

We term this put option the taxpayer put as the losses involved fall on market participants in the general economy who presumably are taxpayers. In this regard we note that in the presence of a bailout by the government, the cost is more uniformly distributed across taxpayers, while in its absence it falls completely on the spectrum of creditor counterparties at time of default. When this put is exercised in the absence of a bailout, one has a wealth transfer to equity holders from creditor counterparties, while with a bailout it is a transfer from taxpayers in general. In the former case, we may speak of a limited liability put or creditor counterparty put while in the latter case we have a taxpayer put. However, given that we do not know ex ante whether a bailout will be involved or not, and noting additionally that the cash flow we model is the same in the two cases, we call this put the taxpayer put, distinguishing it from the Merton put.

The taxpayer put has a strike and maturity that we envisage as follows. The underlying risk is here the level of risky assets net of random and possibly unbounded liabilities. Such exposures cannot and are not permitted to market participants with no reserve capital demonstrating an ability to absorb potential losses. We therefore partition assets on the balance sheet into cash and cash equivalents and the remaining assets. The latter we treat as risky, like in a Merton model with the possibility that they could drop in value to zero. Cash and cash equivalents serve as reserve capital and the taxpayer put comes into the money when risky liabilities net of risky assets rise to amounts exceeding this level of reserve capital.

Alternatively, and equivalently, the taxpayer put may be seen as a put option on risky assets net of risky liabilities but now with a strike that is the negative of the level of reserve capital. Before we consider debt and equity, we note that in the presence of unbounded liabilities, the firm value itself becomes a call option on the spread of risky assets over risky liabilities. The strike is the negative of the level of reserve capital. The maturity of this option is some future time at which liabilities may become too large. The specific date is unclear and we suppose that the market has some future expected test date in mind that varies with market circumstances. This date could in principle reflect some average maturity of outstanding long term debts. We therefore take the maturity of the taxpayer put option as a parameter to be calibrated from market information. This paper is concerned with the valuation of this taxpayer put.

Firms that possess a valuable taxpayer put, have as already noted, an equity value that is a call option even in the absence of debt. There is then an incentive for firms to take on unnecessary risk, that is risks not associated with generating alpha. We go on to design the sensitivity of required capital to unnecessary risk to combat the adverse risk incentives introduced by limited liability. The result is an operational theory for capital requirements that is divorced from any reliance on ratings. Such a contribution could prove useful in the context of the Dodd-Frank

Act that disallows the use of ratings for the setting of capital requirements.

We additionally also deliver a new stock price model as the stock price is now a call option with a strike related to the cash or cash equivalents on hand. The greater the cash on hand the lower is the strike as it is the negative of the cash on hand and the higher is the stock price. Markets, aware of this situation impose a discipline on the stock price. If the market perceives too low a level of cash on hand for the risk exposure entertained, the stock price cannot be maintained short of a capital injection held in cash.

The recognition of such a relationship has led to the call for contingent capital seeking automatic conversions of debt (Flannery (2009)) or Contingent Convertibles (COCOs, Madan and Schoutens (2011)). The need for cash may also be related to the significant drawdowns in revolving credit lines reported for example in Ivashina and Scharfstein (2010). The market value of the support of such cash injections has been independently estimated by Veronesi and Zingales (2010). The motives for increased cash holdings embodied in our model for the stock price concurs with the arguments offered by Bates, Kahle and Stulz (2009) who also document the recent increases in cash holdings by corporations.

The taxpayer put is typically provided via limited liability for free and generally it is not desirable to distribute highly valuable assets for no charge. Recognizing the many benefits of limited liability we do not envisage mechanisms attempting to charge for this taxpayer put option. Nonetheless, as the world advances into many varied contract designs leveraging such access, attention needs to be paid towards ensuring that the associated strikes are sufficiently negative with enough reserve capital at stake that keeps the value of this taxpayer put relatively small. These considerations lead us to present a separate and new methodology for determining the recommended strike or level of reserve capital.[1]

In summary the paper makes two contributions: The recognition and valuation of the taxpayer put option and the presentation of standards for reserve capital backing risk exposures that are divorced from a reliance on ratings. The former employs recent advances (Hurd and Zhou (2010)) in pricing spread options. The second contribution follows Cherny and Madan (2009, 2010) building on the earlier work of Artzner, Delbaen, Eber and

[1] The adoption of such procedures will result in a new financial policy, that we term capital policy, directed towards risk based requisite levels of reserve capital. Other proposals in this direction include Hart and Zingales (2009), who target the CDS rate. The long term costs of such a policy are critical as noted in Campello, Graham and Harvey (2009) and hence they are to be administered with care.

Heath (1999), Carr, Geman and Madan (2001), and Jaschke and Küchler (2001) to operationalize the concept of acceptable risks.

Our strategy for determining both the requisite level of reserve capital and the value of the taxpayer put hinges on inferring the joint law of risky assets and liabilities from market information. Once this joint law is inferred one may do both: Value the taxpayer put and determine the level of required reserve capital. The joint law is inferred from equity option data upon generalizing the Merton model to account for unbounded liabilities and then estimating the resulting compound option model. We perform such valuations for JPMorgan (JPM), Morgan Stanley (MS), Goldman Sachs (GS), Bank of America (BAC), Wells Fargo (WFC) and Citigroup (C), to find both the taxpayer put values and reserve capital levels. The methods could be extended to institutions lacking an option surface using factor analytic representations of their returns on Exchange Traded Funds (ETF's) that have such surfaces but this is left for future research.

Before proceeding with a relatively realistic modeling of the joint risk exposures on the two sides of the balance sheet, we present a simple model in which we take the random component of net assets to be normally distributed and we present the required computations in the context of this simple model. The illustrative simple model reduces all the dimensions of risk exposure to a single number, the volatility of net assets. Its purpose is illustrative. However we do observe in the context of this simple model that debt holder incentives to monitor risk levels are possibly reversed in the presence of the taxpayer put as they are aligned with equity holders with respect to this put option. The effect of cash reserves on the new stock price model are also described in the context of the simple model.

The outline of the rest of the paper is as follows. Section 2 briefly reviews the essentials for computing the required level of reserve capital on behalf of the external economy. Section 3 describes the model for equity as a compound option on the spread of risky assets over liabilities. Computations for the simple model are presented in Section 4. The specific joint process for the correlated evolution of risky assets and liabilities is presented in Section 5. We also provide in this section a description of the balance sheet and option data on the six banks employed in the study. Further the details for calibrating the joint law of risky assets and liabilities along with the calibration results and the computation of required reserve capital, the value of the taxpayer put and related variables of interest are all contained in Section 5. Section 6 concludes.

2 Required Capital Reserves

The section develops and presents closed form expressions for reserve capital for firms exposed to potentially unbounded liabilities. The definition of acceptable cash flows is reviewed with comments on the choice of the base measure with respect to which one defines acceptability. A discussion of the economic foundations for the abstract definition is followed with operational approaches utilizing concave distortions (Cherny and Madan (2009)). Closed form formulas for risk based required capital are the final result.

The set of risks viewed as random variables X on a probability space $(\Omega, \mathcal{F}, P)$ that are acceptable to the general economy are modeled as a cone containing the nonnegative cash flows. We comment later on the choice of the base probability measure P. The inclusion of the nonnegative cash flows is quite natural as these cash flows are always acceptable to anyone, by virtue of being devoid of risk. Thus, this formulation serves as a minimal generalization of accepting just the nonnegative cash flows. We now allow as acceptable, cash flows that may be negative on some contingencies thereby accepting some loss exposure. This loss exposure is accepted in the recognition that demanding a strictly positive cash flow is just too conservative and destructive of growth and innovation in the economy. The extent of losses accepted, however, is controlled by the size of the cone.

It follows as a consequence (see Artzner, Delbaen, Eber and Heath (1999)) that there exists a convex set $\mathcal{M}$ of supporting probability measures $Q \in \mathcal{M}$, equivalent to P, with the property that X is acceptable just if

$$E^Q[X] \geq 0, \text{ for all } Q \in \mathcal{M}, \text{ or} \tag{1}$$

$$\inf_{Q \in \mathcal{M}} E^Q[X] \geq 0. \tag{2}$$

Madan (2009) contrasts this condition with the condition for a cash flow with a positive alpha. For a positive alpha one only needs to require a positive expectation for a single measure Q, representing an equilibrium pricing measure. The class of acceptable cash flows in our model is then generally considerably smaller than just positive alpha cash flows, and the acceptability requirement is thereby considerably more conservative. The larger is the set of supporting measures $\mathcal{M}$ the smaller is the cone of acceptable risks.[2]

[2] We work here with static models for acceptable risks. For dynamic extensions of concepts of acceptable risks we cite Cheridito, Delbaen and Kupper (2004), Riedel (2004), and Roorda, Engwerda and Schumacher (2005).

It is important to remark at this point that the base measure in the original formulations in Artzner, Delbaen, Eber and Heath (1999) and Cherny and Madan (2009) may have been the so called physical or true measure. Such a choice may be of interest to traders, however, from the perspective of generalizing risks acceptable to the general economy, a positive expectation under the physical measure that fails to earn adequate risk compensation may not be acceptable to the general external economy. One may wish to start in such a formulation with a base measure that is already some risk neutral measure. We then expand the set of test measures to add to this base risk neutral measure. In what follows we shall in practice work with a base risk neutral measure to start with that we continue to denote by P.

A risky cash flow may in general not be acceptable as it exposes the general economy to a substantial loss. For example a balanced long short hedge fund going long and short by 100 million dollars must demonstrate some cash capital to be permitted to proceed. A business set up with limited liability, insufficient capital and access to unbounded liabilities if permitted to proceed places too much risk on the general economy. A natural remedy is to seek to add capital in the form of cash at stake in the amount C such that the capitalized firm with cash flow $C + X$ is acceptable.

Hart and Zingales (2009) compare such a magnitude with a margin requirement, or leverage being permitted. We formalize these levels using a formal definition for acceptable risks. It then follows from (2) applied to $C + X$ that the smallest such capital is

$$C = -\inf_{Q \in \mathcal{M}} E^Q[X]. \tag{3}$$

For an already acceptable cash flow satisfying condition (2), this capital required will be negative and one may remove cash and yet be acceptable. When X is not acceptable on its own, as would be the case for a balanced long short fund and a base risk neutral measure, one may use equation (3) to compute the level of reserve capital that the external world demands as a stake in the proposed business.

The role of the mean μ of the cash flow, if any, may now be observed as one may equivalently write that $C + X = C + \mu + (X - \mu)$ to get that

$$C = -\inf_{Q \in \mathcal{M}} E^Q[X - \mu] - \mu. \tag{4}$$

Hence the required reserve capital for the cash flow X is the same as that for the demeaned cash flow or essentially the risky part less the mean. The presence of any mean serves to reduce the required reserve capital. For cash flows with a substantial mean the required reserve capital will then be negative indicative of an already acceptable cash flow.

The proposed methodology for ascertaining whether candidate risk exposures are acceptable to the general economy is broadly consistent with classical utility theory. We may envisage the aggregate risk as being partitioned into N pieces of size $\frac{1}{N}(C+X)$ to be held by N randomly selected persons from the economy. Each of these persons could evaluate the risk using their personalized state price density that in an equilibrium would be a risk neutral density associated with a risk neutral measure Q^i for individual i. By way of examples for such personalized state price densities we cite Telmer (1993), Constantinides and Duffie (1996). Acceptability would then require that $E^{Q^i}[C+X] \geq 0$ for all i. Provided the set $\mathcal{M}$ is large enough to encompass all the measures Q^i the proposed definition of acceptable risks is more conservative than attaining acceptability in a general equilibrium by a wide and partitioned distribution.

In general there are both aggregation and hedging issues in constructing acceptable risks. A random variable X may not be acceptable by itself. However, when it is combined with hedges or other risks that have to be held as an economic endowment, one may find the risk represented by a random variable to be acceptable. In principle the use of personalized measures at the margin allows for the assessment of covariations of risks with endowment risks. For our purposes we suppose we have in X the aggregate risk inclusive of all hedges actually employed.

Alternatively, one could consider acceptability to a single representative utility function $U(x)$. Such an approach leads on marginal analysis to the condition $E[ZX] \geq 0$ for a single candidate reference utility, where Z is classically the normalized marginal utility (Huang and Litzenberger (1988)). The cone of acceptable risks is then too wide in our opinion as it constitutes a half space defined by a single pricing kernel. We comment later on the specific nature of the set $\mathcal{M}$ employed in our definitions for reserve capital.

2.1 Computing the Reserve Capital

The question that now arises is, "How do we compute this required level of reserve capital?". For this we turn to Cherny and Madan (2009).

Suppose as a first approximation that acceptability is defined completely by the probability law or distribution function $F(x)$ of the risk at hand.[3] Cherny and Madan (2009) then describe the link between acceptability and concave distortions of the distribution function. Let $\Psi(u)$ be a concave distribution function on the unit interval and define acceptability as a positive expectation under concave distortion of F by Ψ or the condition

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0. \tag{5}$$

As can be shown (Cherny (2006)), the set of supporting measures $\mathcal{M}$ for this set of acceptable risks is all measures Q with density $Z = \frac{dQ}{dP}$ satisfying the condition

$$E^P\left[(Z-a)^+\right] \leq \Phi(a) =_{def} \sup_{u\in[0,1]} \left(\Psi(u) - ua\right), \text{ for all } a \geq 0. \tag{6}$$

An alternative description of the supporting measures is possible on restricting attention to an original law that is just the uniform distribution on the unit interval. The cash flow is then seen as $F^{-1}(u)$ and for acceptability with respect to Ψ, it must have a positive expectation for all measure changes on the unit interval with density $Z(u)$ for which the corresponding distribution function $H(u)$ with $H' = Z$, is bounded above by $\Psi(u)$. Stated another way, it may be shown that a cash flow is approved by a distortion just if it has a positive expectation for all measure changes on the unit interval whose distribution functions are first order stochastically dominated by the distortion. For further details on this representation see Cherny and Madan (2010).

In summary, the condition (5) defines a valid cone of acceptable risks that depend on just a knowledge of the distribution function of the cash flow. We may rewrite the integral in condition (5), letting $f(x) = F'(x)$, as

$$\int_{-\infty}^{\infty} x\Psi'(F(x))f(x)dx. \tag{7}$$

We then observe that our expectation under concave distortion is also an expectation under a measure change. We note that large losses with $F(x)$ near zero are reweighted upwards by $\Psi'(F(x))$ as Ψ' decreases for any concave distortion. The more concave the distortion the higher the upward reweighting of losses and the more difficult it

[3] We are aware that a focus on just the probability law fails to recognize endowment risk covariations of potential counterparties. The counterparties are however widely distributed across the economy and we focus on the probability law of the risk at hand in defining acceptability as a good first approximation. We note further that such an approach is in agreement with the Sharpe ratio (Sharpe (1964)), the Gain Loss ratio (Bernardo and Ledoit (2000)) and expected utility theory (Huang and Litzenberger (1988)).

is to be acceptable.

Cherny and Madan (2009) go on to propose a sequence of concave distortions indexed by a real number γ that are increasingly more concave with a corresponding decreasing sequence of sets of acceptability. The level γ is the stress level of the distortion and acceptability of a cash flow at stress level γ is a measure of the performance of the cash flow. The recommended distortion that we employ in this paper is *minmaxvar* for which

$$\Psi^{\gamma}(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}. \tag{8}$$

This distortion has the property that large losses associated with u near zero are reweighted upwards towards infinity and $\Psi^{\gamma\prime}(u)$ tends to infinity as u tends to zero. Furthermore as u tends to unity and we have large gains, $\Psi^{\gamma\prime}(u)$ tends to zero whereby large gains are discounted down towards zero as well. The former property reflects loss aversion while the latter is associated with the absence of gain enticement. We present in Figure 1 a graph of this distortion for three values of γ.

A simple computation yields the equation for the required level of reserve capital for stress level γ as

$$C = -\int_{-\infty}^{\infty} x d\Psi^{\gamma}(F(x)) \tag{9}$$

with a computation associated with a simulated set of cash flows sorted into increasing order as $x_1 \leq x_2 \leq \cdots x_N$ by

$$C \approx -\sum_{j=1}^{N} x_j \left(\Psi^{\gamma}\left(\frac{j}{N}\right) - \Psi^{\gamma}\left(\frac{j-1}{N}\right)\right). \tag{10}$$

2.2 Calibrating the recommended stress level

The capital requirement is given by equation (3). We shall see that it is necessary to have capital requirements that are risk sensitive. Consider, especially a change in risk that does not change the risk neutral mean so there is no alpha generated but just an increase in unnecessary risk. Such moves should be associated with an increased capital requirement. Now when the set $\mathcal{M}$ is a singleton with just one element $Q \in \mathcal{M}$ the partial derivative of required capital with respect to unnecessary risk, that is one that leaves the mean unchanged, is zero. Required reserve capital levels are then not sensitive to unnecessary risk.

Suppose now that we have two measures in $\mathcal{M}$. The partial of required reserve capital with respect to risk say the volatility of X, σ_X is now

$$\frac{\partial}{\partial \sigma_X} Max\left(E^{Q_1}[-X], E^{Q_2}[-X]\right) \tag{11}$$

If the maximum is attained at the same measure, say Q^1 for two neighboring levels of risk, and there is no change in the expectation under this measure then this partial is still zero. However, with a change in risk we may have a switch in the measure attaining the maximum and such a move can lead to a higher capital requirement even if the risk neutral mean under Q^1 has not changed. It is therefore important to have many measures in $\mathcal{M}$ to help make required capital sensitive to unnecessary risk. Such considerations dictate the choice of a suitable and minimal stress level γ.

This section has set out the procedure for reserve capital computation once we have the probability law of the risk at hand with a known distribution function. One may then employ equation (9) in a numerical integration. If we only have access to a simulated cash flow then we use equation (10).

3 Equity as a Spread Option

This section presents our extension of the Merton (1973, 1974, 1977) model to now include exposure to unbounded liabilities. When coupled with limited liability for the firm we observe the positive value now given to the taxpayer put. Its existence also distorts debtholder incentives to monitor risk. Furthermore, we observe the cash reserve needs built into the new stock price model. The section closes on describing the procedure for inferring the joint law for the evolution of risky assets and liabilities from data on equity option prices.

3.1 Unbounded Liability Exposures

We modify and extend the model formulated in Gray, Merton and Bodie (2008) that builds on Merton (1973, 1974, 1977). Our point of departure from the context outlined in these papers is access to unbounded liabilities. We account for the access to derivative markets that enables transformations of risk exposures and permits positions in a whole range of contingent and potentially unbounded liabilities. We partition our assets into a reserve capital here taken to be cash Z, risky assets A, with total assets being $A + Z$. The risky component A may fluctuate in value over time.

On the liability side we have a relatively bounded component like risky debt. In addition we allow for risky liabilities that are random and may rise in value, in principle without bound. [4] Hence we have in place of the Mertonian equation with random assets equalling equity plus risky debt, a more general equation

$$\begin{aligned} Cash \ + \ Risky\ Assets &= Equity \ + \ Risky\ Debt \ + \ Risky\ Liabilities \\ Z(t) + A(t) &= J(t) + D(t) + L(t) \end{aligned} \tag{12}$$

The limited liability of equity requires us to recognize that at a market stylized debt maturity T with a face value F, we have that

$$J(T) = (Z(T) + A(T) - L(T) - F)^{+}\,. \tag{13}$$

while debt holders receive

$$D(T) = Min((Z(T) + A(T) - L(T))^{+}\,, F). \tag{14}$$

Incorporating the relative nonrandomness of cash we set $Z(t) = Ze^{rt}$ for a continuously compounded interest rate of r, and we write

$$J(T) = (Ze^{rT} + A(T) - L(T) - F)^{+}. \tag{15}$$

3.2 Equity, Debt, Firm Value, the Taxpayer Put and Reserve Capital

This subsection presents the resulting formulas for the value of equity, debt, firm value and the now positive value for the taxpayer put.

Taking risk neutral expectations of equations (13) and (14) respectively we obtain the value of debt D and equity J in our model. Summing these expressions we obtain the firm value V. Recalling that our base measure P

[4] The risky liabilities could include for example, short positions in stocks, the negative side of swap contracts, payouts on writing credit protections, payouts on selling options or short positions in variance swaps to mention a few possibilities.

is a risk neutral measure, they are given by

$$J = E_0^P\left[e^{-rT}\left(A(T) - L(T) - (F - Ze^{rT})\right)^+\right] \quad (16)$$

$$D = E_0^P\left[e^{-rT}\left(\left(Ze^{rT} + A(T) - L(T)\right)^+ \wedge F\right)\right] \quad (17)$$

$$V = J + D = E_0^P\left[e^{-rT}\left(Ze^{rT} + A(T) - L(T)\right)^+\right]. \quad (18)$$

In addition to the limited liability of equity in equation (16), we now recognize a second limited liability that is the limited liability of the firm itself in equation (18). In the absence of risky liabilities or when $L(T)$ is zero since $A(T) \geq 0$, firm value is positive by construction and the limited liability feature for the firm is redundant. Additionally we may safely take cash reserves Z at zero as we always have a positive value and there is no need for reserve capital requirements.

From the perspective of the stock price in equation (16) we see that the level of cash and cash equivalents on hand reduces the strike and raises the stock price. In the classical model cash on hand impacts stock prices via just its possible impact on volatility.

From the perspective of debt equation (17) we observe that as firm value is now a call option on risky assets less liabilities debt holders participate in this option and their risk attitudes may thereby get aligned with equity holders with little or no incentive to monitor risk as in the classical model.

Recognizing that the firm value of equation (18) is now a call option as the firm now holds the option to put excessive losses back to the general economy we may observe the positive value Y of this option that we call the taxpayer put. By put-call parity

$$\begin{aligned} Y &= E_0^P\left[e^{-rT}\left(-Ze^{rT} - (A(T) - L(T)\right)^+\right] = \\ &\quad E_0^P\left[e^{-rT}\left(Ze^{rT} + A(T) - L(T)\right)^+\right] - Z - (A(0) - L(0)) \quad (19) \\ &\geq 0. \end{aligned}$$

For the acceptability of this risk exposure at unit time by the general economy one must set reserve capital requirements as per equation (9) where we define $F_{A-L}(x)$ to be the distribution function of the law of $A(t) - L(t)$

taken at $t = 1$, and

$$Z^* = -\int_{-\infty}^{\infty} x d\Psi^{\gamma}(F_{A-L}(x)). \tag{20}$$

We recall here that the purpose of imposing reserve capital requirements is to mitigate the adverse risk incentives introduced by limited liability. Madan (2009) shows that for an all equity firm if risk based reserve capital requirements are sufficiently conservatively set, with a high enough stress level γ, then the positive partial derivative of equity value with respect to risk can be offset by the positive and larger sensitivity of required reserve capital to risk, yielding a negative aggregate partial derivative for profits with respect to risk.

In the presence of debt it is well known from the Merton model that equity holders will want to increase volatility to maximize the post debt value of the call option held by equity while debt holders may wish to write contracts attempting to hold down this volatility. With the additional presence of the taxpayer put option however, debt holders now have an incentive to raise volatility as they participate in the taxpayer put though they have sold a put at a higher strike to equity holders. The net effect of volatility on the value they receive is now ambiguous. We comment further on these issues in the context of the simple model of the next section.

Furthermore as both debt holders and equity holders jointly receive the benefits of the taxpayer put the question arises as to whether debt holders will enforce higher levels of reserve capital on their own. These questions are the subject matter of a separate study with the results possibly depending on how contributions to additional reserve capital are to be shared between debt holders and equity holders. Our interest is focused on valuing the taxpayer put and determining the level of required capital reserves Z^*.

3.3 Procedure for calibrating the joint law of assets and liabilities

We model the risk neutral law of $A(T) - L(T)$ as the difference of two exponential Lévy processes and we have in closed form the joint characteristic function for the logarithm of the asset and liability levels. We may therefore evaluate at any date t, given the prevailing level of the pair (A_t, L_t) the value of equity at time t, $J(t)$, as an option on this spread.

Specifically, this computation is a two dimensional Fourier inversion for the equity value seen as a convolution of the spread option payoff and the joint density for assets and liabilities (Hurd and Zhou (2010), Madan (2009)).

Specifically we have

$$J(t) = E_t^P \left[e^{-r(T-t)} \left(A(T) - L(T) - (F - Ze^{rT}) \right)^+ \right]. \tag{21}$$

The particular times t of interest are the maturities of traded equity options for which we have data from the equity options market. These time points are typically below two years with up to ten traded maturities. To access potential levels of the pair (A_t, L_t) at the equity option maturities we simulate forward for two years the probability law for the joint process of assets and liabilities. On this joint path space for assets and liabilities we evaluate at the equity option maturities the price of equity as a spread option as per equation (21). We thus obtain a matrix of simulated equity prices at the equity option maturities. We use this matrix to estimate equity option prices $w(K,t)$, for strike K and maturity t as

$$w(K,t) = e^{-rt} E_0^P \left[(J(t) - K)^+ \right], \tag{22}$$

by averaging over the simulated realizations of the equity values for each of the required maturities. The parameters of the joint and correlated risky asset and liability value process are determined to best fit the surface of the equity option prices as seen on the option markets.

We then compute the value Z^* as per equation (20). We set the stress level γ at 0.75 as suggested in Madan (2009) as a minimal level mitigating the adverse risk incentives introduced by limited liability. We compare this required level of reserve capital with the level obtained from balance sheets to determine which banks were undercapitalized or overcapitalized from the perspective of risk exposure to the external economy. We also compute the value of the taxpayer put as per equation (19). The value of Z^* is determined at an annual maturity. For the taxpayer put value we use the maturity provided by equation (21) at time $t = 0$.

In summary, this section has presented the implications of exposure to unbounded liabilities for firms granted limited liability. A special role for cash reserves has been observed in the equation for the stock price. It is noted that debt holders are partially aligned with stock holders. The firm holds a taxpayer put with a positive value. Finally a revised spread option model for equity leads to procedures for identifying the joint law of assets and liabilities from data on equity option prices.

4 An Illustrative Model

This section presents a simple model in which net assets are a Gaussian random variable. In this context we first evaluate explicitly the value of the taxpayer put, and its sensitivity to cash reserves. This is followed by a closed form formula for risk sensitive levels for required reserve capital. We then take up the risk monitoring incentives of debt holders and the role of cash reserves in the new stock price model. It is observed that especially with lower cash reserves debt holders may no longer monitor risk and stock prices may get too far out of the money.

4.1 A simple model where net assets are Gaussian

Consider the case of a balanced long short hedge fund with access to a risky cash flow X. As a simple model we suppose that X is normally distributed with mean μ_X and variance σ_X^2. If we take the notional level of assets and liabilities at N and take a percentage volatility of σ for both the assets and liabilities then with a correlation of ρ one would estimate

$$\sigma_X = \sqrt{2}\sigma N\sqrt{1-\rho}. \tag{23}$$

Proposition 1. For a notional of N with debt face value F, debt maturity T, cash reserves Z, an interest rate r, percentage volatility σ, asset liability correlation ρ, volatility of net assets X of σ_X as per equation (23) and mean μ_X, the values of equity J, debt D, the firm V, the required reserve capital Z^*, the taxpayer put Y and its derivative with respect to reserves are

$$J = \frac{\sigma_X e^{-rT}}{\sqrt{2\pi}} \exp\left(-\frac{(F - Ze^{rT} - \mu_X)^2}{2\sigma_X^2}\right) - \left(Fe^{-rT} - Z - \mu_X e^{-rT}\right)\Phi\left(-\frac{F - Ze^{rT} - \mu_X}{\sigma_X}\right). \tag{24}$$

$$V = \frac{\sigma_X e^{-rT}}{\sqrt{2\pi}} \exp\left(-\frac{(Ze^{rT} + \mu_X)^2}{2\sigma_X^2}\right) + \left(Z + \mu_X e^{-rT}\right)\Phi\left(\frac{Ze^{rT} + \mu_X}{\sigma_X}\right). \tag{25}$$

$$D = V - J. \tag{26}$$

$$Z^* = A(\gamma)\sqrt{2}\sigma N\sqrt{1-\rho} - \mu_X. \tag{27}$$

$$Y(Z) = \frac{\sigma_X e^{-rT}}{\sqrt{2\pi}} e^{-\frac{\left(Ze^{rT}+\mu_X\right)^2}{2\sigma_X^2}} - \left(Z + e^{-rT}\mu_X\right)\Phi\left(-\frac{Ze^{rT} + \mu_X}{\sigma_X}\right). \tag{28}$$

$$Y'(Z) = -\Phi\left(-\frac{Ze^{rT} + \mu_X}{\sqrt{2}\sigma N\sqrt{1-\rho}}\right) \tag{29}$$

where $\Phi(x)$ is the distribution function for a standard normal variate and $A(\gamma) = -\int_0^1 \Phi^{-1}(u)\Psi^{\gamma\prime}(u)du$ is a reweighted integral of the inverse of the standard normal distribution function.■

We first observe that any presence of a mean cash flow is merely added to the strike after discounting. The underlying risk can then be seen as one with a zero expectation. At $Z = \mu_X = 0$ the value of this taxpayer put is

$$Y(0) = \frac{\sqrt{(1-\rho)}e^{-rT}}{\sqrt{\pi}}\sigma N,$$

and this can be quite substantial for assets and liabilities not perfectly correlated and a high notional. For example with a 100 million dollar notional, a volatility of 10% with correlation 25%, a five year put for an interest rate of 5% is valued at 3.4158 million dollars. Were the volatility to increase to 15% the value of the put rises to 5.3185 million dollars. We see from equation (29) that the value of this taxpayer put declines as one raises the level of reserve capital and approaches zero as Z tends to infinity.

With regard to the required level of reserve capital we may evaluate expression (27). We see that the required level of capital reserves net of the mean payout are proportional to volatility with the factor of proportionality depending on the stress level employed. For minmaxvar at level .75 this factor of proportionality is 1.07. It falls to .75 for a stress level of .5 and rises to 1.35 for a unit stress level. For a 10% volatility with correlation 25% on a 100 million notional and a zero mean the required reserve capital is 13.0447 million. The required reserve capital is reduced by any mean that may be present. So if the mean is raised to 10 million then the required reserve capital is reduced to 3.0447 million and for a mean of 15 million the required reserve capital is -1.9553 million. On the other hand if the mean is -10 million the required reserve capital rises to 23.0447 million.

We next ask if debt holders retain in the presence of the taxpayer put an incentive to reduce volatility. For a face value of F, debt holders hold a call struck at $-Ze^{rT}$ less a call struck at $F - Ze^{rT}$. We present in Figure 2 graphs of the value of this call spread as a function of volatility in the two cases, $Z = 0$ and for $Z = 40$. The computation uses the base case of $N = 100$, $r = 5\%$, $T = 5$, $\mu = 0$ and $\rho = .25$ for a debt face value of $F = 50$.

We observe that if no reserve capital is required then debt holders have lost their incentive to hold down volatility, benefiting more from the taxpayer put than they lose from equity holders putting losses back to them. However, if significantly high capital reserves are imposed at $Z = 40$, then at low volatility levels debt holders retain some classical incentives to monitor risk downwards but at higher volatility levels they work towards raising volatility.

With respect to the incentives on the part of debt holders to insist on required capital reserves we present in Figure 3 the profit on debt measured as the debt value less the share of required reserves provided by debt holders, assuming they contribute equally to reserves. We observe that at low reserve levels debt holders receive some benefit from increased reserves while equity holders would be opposed to such an increase. It is unclear if one can rely on debt holders to effectively participate in risk monitoring in the presence of the taxpayer put. Externally imposed risk based capital requirements may be what is needed.

Finally we address the role of cash on hand on the stock price. A significant drop in cash on hand unaccompanied by an increase in the risk neutral mean will result in a drop in the stock price that may only be protected by the injection of capital held as cash. The sensitivity of stock prices to volatility is also enhanced.

4.2 Calibrating stress levels

Limited liability makes the firm value a call option by delivering to the firm the taxpayer put. This makes firm value positively sensitive to unnecessary risk. Capital requirements should be sufficiently risk sensitive to counter this positive sensitivity of firm value to unnecessary risk. In the simple model we observe from equation (27) and (23) that

$$\frac{\partial}{\partial \sigma_X} Z^* = A(\gamma) \tag{30}$$

On the other hand for a zero mean and zero rates the partial derivative of firm value with respect to risk may be evaluated as

$$\frac{\partial}{\partial \sigma_X} V = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Z^2}{2\sigma_X^2}\right) \tag{31}$$

If we take Z at Z^* as per equation (27) and we equate the sensitivity of capital to the sensitivity of firm value we obtain that the stress level γ should satisfy

$$\exp\left(-\frac{A(\gamma)^2}{2}\right) = \sqrt{2\pi} A(\gamma) \tag{32}$$

We may observe that equation (32) implies that

$$.3722 = A(\gamma). \tag{33}$$

The value of γ is then computed to be 0.2222 for the simple model. Similar exercises may be conducted in other more realistic contexts to motivate the choice of an appropriate and minimal stress level.

Thus, this section has illustrated in the context of a simple model the positive value for the taxpayer put, the required level of risk based reserve capital, and the implications of unbounded liabilities for the value of debt along with the effects of cash reserves on stock prices.

5 Joint Process for Assets and Liabilities, Data, Calibrations and Results

This section presents a more realistic model for the joint evolution of assets and liabilities, building on past successes in the literature calibrating option surfaces. The simple model of section 4 is counterfactual and not consistent with market data on option prices. The more realistic model of this section is designed to capture various dimensions of risk and employs factors to allow for both compensating and jointly adverse moves to both sides of the balance sheet. A description of the joint law is followed by the description of the data employed, the exact calibration procedure used and finally the results for the value of the taxpayer put, reserve capital and related variables of interest.

5.1 The Joint Law of Assets and Liabilities

For a more realistic evaluation using data on equity option prices we build on the literature calibrating option pricing models. For a single maturity it is now fairly well established that a number of exponential Lévy process models adequately describe option prices across strikes. Examples include the variance gamma model (Madan and Seneta (1990) and Madan, Carr and Chang (1998)). Extensions to the $CGMY$ model (Carr, Geman, Madan and Yor (2002)) are an alternative. Other possibilities include the hyperbolic and generalized hyperbolic models, Eberlein and Keller (1995), Eberlein (2001) and Eberlein and Prause (2002) and the normal inverse Gaussian model (Barndorff-Nielsen (1998)).

When it comes to pricing options across maturities as well as strikes, Konikov and Madan (2002) report on the inadequacy of a Lévy process in this regard. However it is also known that option prices are free of static arbitrage provided they are consistent with a one dimensional Markov model (Carr, Geman, Madan and Yor (2003) and Carr and Madan (2005)). This led to the development of the Sato process that successfully calibrated option surfaces with a four parameter one dimensional Markov model, in fact the Sato process is an additive process or process with independent but inhomogeneous increments for the logarithm of the stock price (Carr, Geman, Madan and Yor (2007)).

For both the Lévy and Sato process models for the underlying stock price under the physical measure, the market is known to be incomplete and we do not have a unique price for options nor do we know that the risk neutral law corresponds to a Lévy or Sato process. When using these methods, we generally do not specify the physical law and there is no replication strategy given the incompleteness, instead we propose a parametric model in this class for the risk neutral law. The actual estimated parameter values are then inferred from observed market option prices. In the current context the underlying risks are given by the joint law for the assets and liabilities of the firm that are presumed to have a risk neutral law that we model directly.

With a view to keeping things relatively simple given that we have to calibrate a compound option model by simulation we take an intermediate approach and model our risky assets and liabilities as exponential Lévy processes with

$$A(t) = A(0)\exp\left(X(t) + (r+\omega_X)t\right) \tag{34}$$

$$L(t) = L(0)\exp\left(Y(t) + (r+\omega_Y)t\right). \tag{35}$$

With a view towards modeling dependence in these processes we consider linear mixtures of independent Lévy processes. Such joint laws have been considered in the time series context by Madan and Yen (2007), Madan (2006) and Khanna and Madan (2009). In the time series applications the required mixing matrix is estimated using independent components analysis (Hyvärinen, Karhunen, and Oja (2001)). For our risk neutral application we design the mixing matrix exogenously.

If we were to take a linear mixture of just two independent Lévy processes we would get jumps occurring on two rays from the origin. If the independent processes are variance gamma VG processes for example then we have a VG process running in log space on a particular ray from the origin with the asymmetry parameter on this ray being the skewness parameter of the VG distribution. The VG process uses three parameters for each ray which is two sided. Given that we operate in a two sided way for each independent Lévy process, we need to cover 180 degrees of possible directions of motion. Somewhat more generally we take 4 VG processes with 12 parameters placed at the degrees 30, 60, 120, and 150. This gives us two rays with a positive relation between asset and liability movements and two rays with a negative dependence. We shall let the calibration determine the relative variance placed on each of the four rays. For the four angles η_j, $j = 1, \cdots 4$ we have the jumps in assets and liabilities as

$$x_j = u_j \cos(\eta_j) \tag{36}$$

$$y_i = u_j \sin(\eta_j) \tag{37}$$

where u_i is the jump in the j^{th} VG process with parameters $\sigma_j, \nu_j, \theta_j$. We then have that

$$\begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} \cos(\eta_1) & \cos(\eta_2) & \cos(\eta_3) & \cos(\eta_4) \\ \sin(\eta_1) & \sin(\eta_2) & \sin(\eta_3) & \sin(\eta_4) \end{bmatrix} \begin{bmatrix} U_1(t) \\ U_2(t) \\ U_3(t) \\ U_4(t) \end{bmatrix} \tag{38}$$

and our joint law is the linear mixture of 4 independent VG Lévy processes with a prespecified mixing matrix.

The pricing of equity as a call option on the spread of assets over liabilities requires access to the joint probability law of risky assets and liabilities. This is now accomplished via the joint characteristic function.

Proposition 2. The joint characteristic function for $(\ln(A(t)), \ln(L(t)))$ under the model (34),(35) and the factor representation (38) is given by

$$E\left[e^{iu\ln(A(t))+iv\ln(L(t))}\right] = \phi(u,v)\exp(iu\ln(A(0)) + iv\ln(L(0)) + iu(r+\omega_X)t + iv(r+\omega_Y)t), \tag{39}$$

where $\phi(u,v)$ is the joint characteristic function of $(X(t),Y(t))$ with

$$\phi(u,v)=\prod_{j=1}^{4}\left(\frac{1}{1-i(u\cos(\eta_j)+v\sin(\eta_j))\theta_j\nu_j+\frac{\sigma_j^2\nu_j}{2}(u\cos(\eta_j)+v\sin(\eta_j))^2}\right)^{\frac{t}{\nu_j}} \tag{40}$$

and

$$\omega_X = \sum_{j=1}^{4}\frac{1}{\nu_j}\ln\left(1-\cos(\eta_j)\theta_j\nu_j-\frac{\sigma_j^2\nu_j\cos^2(\eta_j)}{2}\right) \tag{41}$$

$$\omega_Y = \sum_{j=1}^{4}\frac{1}{\nu_j}\ln\left(1-\sin(\eta_j)\theta_j\nu_j-\frac{\sigma_j^2\nu_j\sin^2(\eta_j)}{2}\right).\blacksquare \tag{42}$$

Our equity value at any date t given a simulation of $A(t), L(t)$ is the price of a spread option with some strike and maturity using this joint characteristic function with initial values $A(t), L(t)$ and time to maturity $T-t$. For the initial value of risky assets and risky liabilities excluding debt, we take these magnitudes from the balance sheet but permit some option market adjustment factor to match the stock price. The adjustment factor is calibrated by equating the value of equity computed as a spread option at the strike of debt less cash equivalent reserves with the initial stock price at market close on the calibration date. We are essentially inferring the risk neutral mean for risky assets and liabilities as seen by the initial stock price.

5.2 Balance Sheet and Option Data

For the balance sheet we access Compustat data from Wharton Research Data Service. For each of the six banks we obtained data for the year end 2008. We first take data on cash plus short term investments, the variable CHE in Compustat and we shall use this value for our initial reserve capital level or the variable Z in our calibration procedures. For risky assets, A, we take total assets, AT in Compustat less CHE. For risky liabilities, L, we take all liabilities less the sum of long term debt ($DLTT$) and debt in current liabilities (DLC). For the level of debt, D, we take $DLTT$ plus DLC. In addition we need the number of shares outstanding, n, and the stock price, S. The data is presented in Table 1.

The other data we shall bring to bear on the study of required reserve capital levels is the option surface at year end. Here we have over a hundred options trading at any time. We present along with the calibration results later a sample graph in Figure (4) of the market option prices used in the calibration along with the fitted prices from the compound spread option model. We take option maturities below 1.5 years.

5.3 Calibration Details

For each of the six banks we take data on equity option prices at the date of the balance sheet statement and we describe details for JPM. The level of risky assets was 1806.9 billion and risky liabilities were at 1009.28 billion. The number of shares was 3732 million. We define $A(0)$, $L(0)$ to be risky assets and liabilities on a per share basis at 484.16 and 270.44 respectively. The total debt was at 633.47 billion and the value of Z was 368.15 billion and this gives us a strike on a per share basis of $(633.47 - 368.15)/3.732 = 71.0932$. Technically the strike should be future valued to the maturity but given the low rates and relatively short maturities involved we ignored this adjustment to the strike. The stock price was 31.59.

We take as parameters the maturity of equity as a spread option on the underlying spread of assets over liabilities and the 12 VG parameters on the four rays on which we run our mixture of VG processes. The first step is to solve for ξ such that the value of the spread option starting at asset level $A(0) * (1 - \xi)$, and liability level $L(0) * (1 + \xi)$ equals the observed market stock price of 31.59. The calibration of ξ essentially sets the option implied level for the risk neutral mean raising the strike for the taxpayer put and reducing the required reserve capital by this magnitude. The adjustment of initial assets and liabilities is done for the chosen set of VG parameters and ξ is the option market adjustment factor for the risk neutral mean.

The next step is to generate paths of assets and liabilities daily for 1.5 years and we generated 10000 such paths. Then we use the spread option pricing algorithm to compute a grid of prices of equity as a spread option at all the maturities for which we have equity option data. This grid is used to interpolate equity values for each of the maturities and all the 10000 paths. Given the interpolated equity values we compute the prices of equity options at all the traded strikes and maturities for which we have option data. We then form the mean square error between observed market option prices and the model computed option prices. This procedure gives a single value for the objective function that is minimized by an optimizer over the 13 dimensions of the 12 VG parameters and the maturity of the equity as a spread option.

5.4 Calibration Results

We report the results in the order JPM, MS, GS, BAC, WFC and C. The estimated maturities for equity as a spread option were close to 5 years and are explicitly 4.4726, 4.9890, 5.0036, 5.0025, 4.9893, and 4.9991. We report the VG parameters for the four angles in two separate tables (Table 2 and 3) along with the implied levels of volatility, skewness and kurtosis for the activity at each angle for the Lévy process taken at three months.

We observe that skewness is negative on the positive angles and positive on the negative angles. Hence down jumps are more likely in directions where the factors move together, while up jumps are more likely when they move in opposite directions. The negatively skewed shocks induce positive correlation between assets and liabilities inducing a common negative tail to both assets and liabilities. There are also positively skewed shocks that in our model simultaneously reduce asset values and raise the value of liabilities (note the cosine of 120 is negative while the sine is positive). The strong upward skews at 120 degrees have this simultaneous effect of adversely impacting both sides of the balance sheet. Simple models of dependence like Gaussian correlations essentially place all activity on a single line and do not allow for varied impacts on different occasions. We have different levels of activity displaying different types of dependence taking place at different times. From the volatility estimates we observe that most activity takes place at the 60^0 and 120^0 angles.

We present in Figure 4, by way of a sample the observed and fitted option prices for JPM. The fit is not as good as one expects from the use of a Sato process, but as already commented we are aware of this and have used the simpler Lévy processes to illustrate our methods.

5.5 Computations of Required Reserve Capital Levels and the Value of the Firm's Limited Liability Put

We present in Table 4 the computed externally required reserve capital levels at the stress level of 0.75 that was recommended in Madan (2009) for the distortion *minmaxvar*. Also presented are the level of cash equivalent capital held, the value of the taxpayer put held by the firm, the ratio of required reserve capital to cash equivalent capital held and the option adjustment factor.

We observe that GS has a negative required capital reserve of 84 billion dollars and from our simple model this is indicative of a high level for the market inferred risk neutral mean level of cash flows. We may apply our adjustment factors to the levels of risky assets and liabilities from Table 1, deflating the assets and inflating the liabilities and subtracting the adjusted liabilities from the adjusted assets to get values for the mean cash flows for each bank.

These values are in billions of dollars 46.05, 61.98, 190.49, 184.80, 76.13 and 77.14 respectively for JPM, MS, GS, BAC, WFC and C. As a percentage of the notional the values are 2.55, 13.82, 29.76, 10.92, 6.15 and 4.78. Clearly GS stands out with a mean cash flow estimated at 30% well above the other banks. For the other banks, excluding MS, additional cash equivalent reserves are required. MS could reduce its cash equivalent holdings by around 50%. The others need to add cash capital with WFC facing the biggest shortfall. The shortfall in the case of WFC may just be a consequence of having recently taken over Wachovia. The value of the taxpayer put is low for GS and MS with much higher values for JPM and WFC and intermediate values for BAC and C.

6 Conclusion

Exposure to potentially unbounded liabilities by limited liability firms introduces novel features into financial analysis. Even all equity firms now hold a call option having received a free option to put excessive losses back to the general economy or taxpayers. This option is here termed the taxpayer put and its strike is the negative of the level of reserve capital at stake in the firm. Our contribution is to value this taxpayer put and to determine an appropriate required level of reserve capital. The required capital is determined without reference to ratings and this is an advantage given the position of the Dodd-Frank Act to minimize the reliance of financial calculations on ratings. The required capital is designed to combat the adverse risk incentives introduced by limited liability at the firm level as the taxpayer put delivers an unhealthy appetite for unnecessary risk. Simple closed forms are developed for both the value of the taxpayer put and the required level of reserve capital in the context of a simple Gaussian model when risky assets net of risky liabilities are normally distributed. Furthermore, in the context of this simple model it is observed that the presence of the taxpayer put may reverse the usual debt holder incentives to monitor risk. Participation in the taxpayer put induces debt holders to be partially aligned with equity holders.

The taxpayer put also delivers a new stock price model in which stock prices are call options with strikes that are reduced by the cash reserves on hand, thereby raising the stock price. Markets may then impose a stock price discipline on banks perceived to be short of cash reserves given their risk exposure. It may then become necessary to recapitalize banks with new equity that necessarily has to be held in cash with little ability to grow the balance sheet.

More realistically risky assets and risky liabilities are modeled as two correlated, positive random processes. We take them to be exponentials of two processes that are themselves modeled as linear mixtures of independent Lévy processes where the latter may be viewed as factors. Some of these factors drive assets and liabilities with positive covariations while the others induce negative covariations. Hence, we allow for the presence of independent shocks that simultaneously adversely affect both sides of the balance sheet while other shocks affect assets and liabilities in a compensating way. As a consequence equity becomes a call option on the spread of risky assets over risky liabilities. We employ recently developed methods to value these spread options using a two dimensional Fourier inversion.

The required level of reserve capital is computed as suggested in Madan (2009), with a view to making the aggregate risk of risky assets less risky liabilities acceptable to the external economy. The specific form of acceptability employed is a positive expectation after distortion of the distribution function accessed. Such distortions evaluate expectations by exaggerating losses and discounting gains. This procedure is in accordance with Basel 3 reform plans for regulatory capital requirements. These plans ask for the inclusion of market data from stressed periods when computing reserve capital. The distortion employed is *minmaxvar* as introduced in Cherny and Madan (2009) and the stress level used is 0.75. To work out the required level of reserve capital and the value of the taxpayer put option, we infer the risk neutral law for the joint evolution of risky assets and liabilities from the equity option surface.

The estimated risk neutral process is then used to determine externally required reserve capital levels and implicit taxpayer put option values. We find GS and MS to be sufficiently capitalized with also lower taxpayer put option values, while the other four banks are undercapitalized with the greatest shortfall occurring for WFC. The taxpayer put option values for the other banks are also quite substantial.

The methods developed here may also usefully be employed to analyze the capital requirements and implicit valuations for the taxpayer put for government sponsored entities like Fannie Mae and Freddie Mac. The requisite option data is available. One would however need to consider the nature of the special provisions for government sponsorship in the formulation of the model. There may be an implicit strike to be inferred from market data.

A more aggregative analysis could also be attempted perhaps using data for options on financial sector exchange traded funds with a view to gauging the level of systemic risk. One could model all banks together as exposed to the common factor of the exchange traded fund plus an idiosyncratic factor with a partitioning of total risk charges into a systemic component plus an idiosyncratic one. The former could serve as a contribution to the FDIC for covering systemic risk exposures while the latter is held as a capital reserve at the bank level. The explicit details for such a decomposition are questions for future research.

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Appendix

Proof of Proposition 1. The stock price is given by

$$\begin{aligned} J &= e^{-rT} E_0^P \left[\left(X - (F - Ze^{rT}) \right)^+ \right] \\ &= \frac{\sigma_X e^{-rT}}{\sqrt{2\pi}} \exp\left(-\frac{(F - Ze^{rT} - \mu_X)^2}{2\sigma_X^2} \right) - \left(Fe^{-rT} - Z - \mu_X e^{-rT} \right) \Phi\left(-\frac{F - Ze^{rT} - \mu_X}{\sigma_X} \right). \end{aligned}$$

The value of the firm V is given by the formula for equity value taken at $F = 0$. The value of debt D is given by

$$\begin{aligned} D &= e^{-rT} E_0^P \left[\left(X + Ze^{rT} \right)^+ \wedge F \right] \\ &= V - J \end{aligned}$$

The value of the taxpayer put is given by

$$\begin{aligned} Y(Z) &= e^{-rT} \int_{-\infty}^{-Ze^{rT}} \left(-Ze^{rT} - x \right) \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left(-\frac{(x - \mu_X)^2}{2\sigma_X^2} \right) dx \\ &= \frac{\sigma_X e^{-rT}}{\sqrt{2\pi}} e^{-\frac{\left(Ze^{rT} + \mu_X \right)^2}{2\sigma_X^2}} - \left(Z + e^{-rT} \mu_X \right) \Phi\left(-\frac{Ze^{rT} + \mu_X}{\sigma_X} \right) \end{aligned}$$

The required reserves are given by

$$\begin{aligned} Z^* &= -\int_{-\infty}^{\infty} x d\Psi^\gamma \left(\Phi\left(\frac{x - \mu_X}{\sigma_X} \right) \right) \\ &= -\sigma_X \int_{-\infty}^{\infty} z d\Psi^\gamma(\Phi(z)) - \mu_X \\ &= -\sigma_X \int_0^1 \Phi^{-1}(u) \Psi^{\gamma\prime}(u) du - \mu_X \\ &= A(\gamma) \sqrt{2} \sigma N \sqrt{1 - \rho} - \mu_X \end{aligned}$$

where $A(\gamma)$ is just a reweighted integral of $\Phi^{-1}(u)$.

Proof of Proposition 2.

The joint characteristic function of $\ln(A(t)), \ln(L(t))$ is given by

$$\begin{aligned} E\left[\exp\left(iu\ln(A(t)) + iv\ln(L(t))\right)\right] &= \exp(iu\ln(A(0)) + iv\ln(L(0)) + iu(r+\omega_X)t + iv(r+\omega_Y)t) \\ &\quad \times E\left[\exp\left(iuX(t) + ivY(t)\right)\right] \end{aligned}$$

Now $(X(t), Y(t))$ are linear combinations of independent VG Lévy processes with

$$iuX(t) + ivY(t) = \sum_j (iu\cos(\eta_j) + iv\sin(\eta_j))U_j(t).$$

It follows that

$$\begin{aligned} \phi(u,v) &= E\left[\exp\left(iuX(t) + ivY(t)\right)\right] \\ &= \prod_{j=1}^{4} \left(\frac{1}{1 - i(u\cos(\eta_j) + v\sin(\eta_j))\theta_j\nu_j + \frac{\sigma_j^2\nu_j}{2}(u\cos(\eta_j) + v\sin(\eta_j))^2} \right)^{\frac{t}{\nu_j}} \end{aligned}$$

The values for ω_X, ω_Y follow on evaluating the negative of the logarithms of $\phi(-i,0)$, $\phi(0,-i)$ respectively for $t = 1$, to ensure a risk neutral growth rate of r.

TABLE 1

Balance Sheet on 6 Banks at end of 2008

	Cash, Z	Assets, A	Liab., L	Debt, D	No. of Shares	Stock Price
		in billions of dollars			millions	dollars
JP Morgan, JPM	368.15	1806.90	1009.28	633.47	3732	31.59
Morgan Stanley, MS	210.52	448.29	181.16	392.27	1047	15.16
Goldman Sachs, GS	244.43	640.12	298.55	498.42	443	82.24
Bank of America, BAC	124.91	1693.04	883.00	632.95	5017	13.93
Wells Fargo, WFC	72.09	1237.55	781.40	375.23	4228	29.86
Citigroup, C	325.68	1612.79	769.57	720.32	5450	6.88

TABLE 2

Model is Linear Mixture of 4 independent VG processes

VG Process at 30^0

	σ	ν	θ	$Vol.$	$Skew$	$Kurt.$
JPM	0.0955	0.1558	−0.0178	.0958	−.1735	4.8897
MS	0.0476	0.1491	−0.0593	.0528	−.9414	5.3985
GS	0.0018	0.1509	−0.0434	.0170	−1.5538	6.6214
BAC	0.0289	0.1490	−0.0474	.0342	−1.1207	5.6648
WFC	0.0385	0.1594	−0.0476	.0429	−.9911	5.5889
C	0.0553	0.1501	−0.0505	.0578	−.7466	5.1797

VG Process at 60^0

JPM	0.4018	0.0810	−0.8448	.4682	−.7998	4.4170
MS	0.1422	0.0843	−0.1927	.1528	−.6093	4.2646
GS	0.1605	0.0937	−0.1935	.1711	−.6105	4.3778
BAC	0.0958	0.0744	−0.1792	.1075	−.6926	4.2235
WFC	0.0735	0.0875	−0.2037	.0950	−.9745	4.7244
C	0.1990	0.1007	−0.2001	.2089	−.5610	4.4214

TABLE 3

VG Process at 120^0

	σ	ν	θ	$Vol.$	$Skew$	$Kurt.$
JPM	0.0968	0.1778	0.2967	.1582	1.5837	6.9680
MS	0.1699	0.2693	0.3217	.2382	1.8249	8.6727
GS	0.0761	0.2133	0.2092	.1230	1.7291	7.7440
BAC	0.0016	0.2331	0.2757	.1331	1.9312	8.5944
WFC	0.1088	0.2564	0.3439	.2053	1.9589	8.9111
C	0.1098	0.1992	0.2016	.1420	1.4700	6.9253

VG Process at 150^0

JPM	0.0116	0.3524	0.0175	0.0118	.1028	3.1463
MS	0.0240	0.2003	0.0522	0.0253	.2867	3.3437
GS	0.0117	0.2002	0.0072	0.0117	.0430	3.1416
BAC	0.0671	0.2175	0.0737	0.0698	.4147	3.9213
WFC	0.0105	0.2023	0.0614	0.0122	.2882	3.1838
C	0.0598	0.1999	0.0209	0.0600	.1246	3.7280

TABLE 4

In Billions of Dollars

	Required Reserve Capital Levels	Reserve Capital Held	Limited Liability Put Value	Required to Actual Ratio	Adjustment Factor
JPM	698.04	368.15	293.96	1.8961	0.3154
MS	116.27	210.52	29.75	0.5523	0.4113
GS	−83.84	244.43	3.37	−0.3430	0.1796
BAC	246.07	124.91	158.17	1.9700	0.2840
WFC	366.83	72.09	220.14	5.0884	0.2107
C	434.60	325.68	156.21	1.3344	0.3984

This Table displays for the six major US banks as at the end of 2008 the required reserve capital, the value of the taxpayer put, and the adjustment factor calibrating the market stock price. Also shown are the reserves held and the ratio of required to actual reserves. The computations are based on the risk neutral model for assets and liabilities seen as a linear mixture of four independent variance gamma processes.

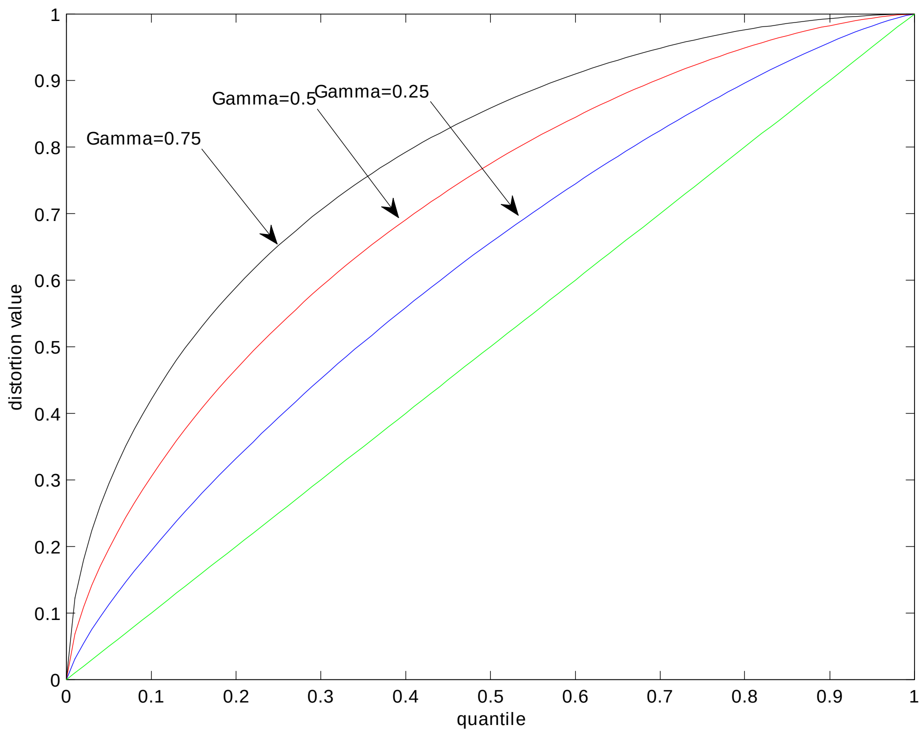

Figure 1: Graph of the distortion minmaxvar for three settings of the stress level Gamma. The levels are .25, .5 and .75. The derivatives at zero and unity are infinite and zero respectively.

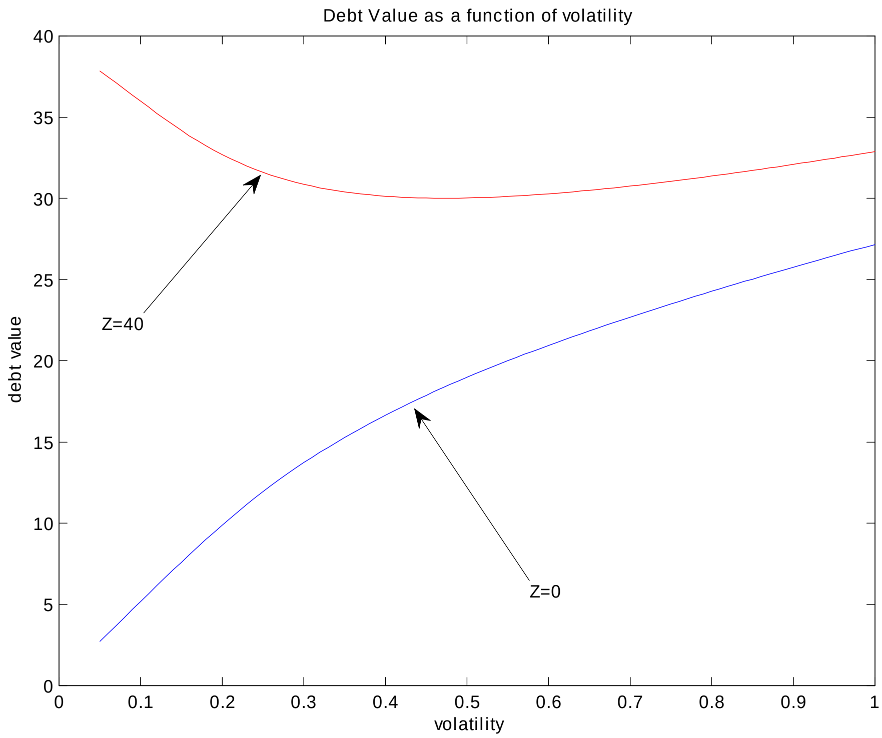

Figure 2: Debt Value as a function of volatility in the presence of the taxpayer put at two levels of required reserve capital. The value is computed to reflect the fact that debtors now hold a put spread and so the value may both rise and fall with volatility depending on the two strikes. For Z=0 its is an increasing function and incentives to monitor risk have disappeared. For Z=40 it is U shaped.

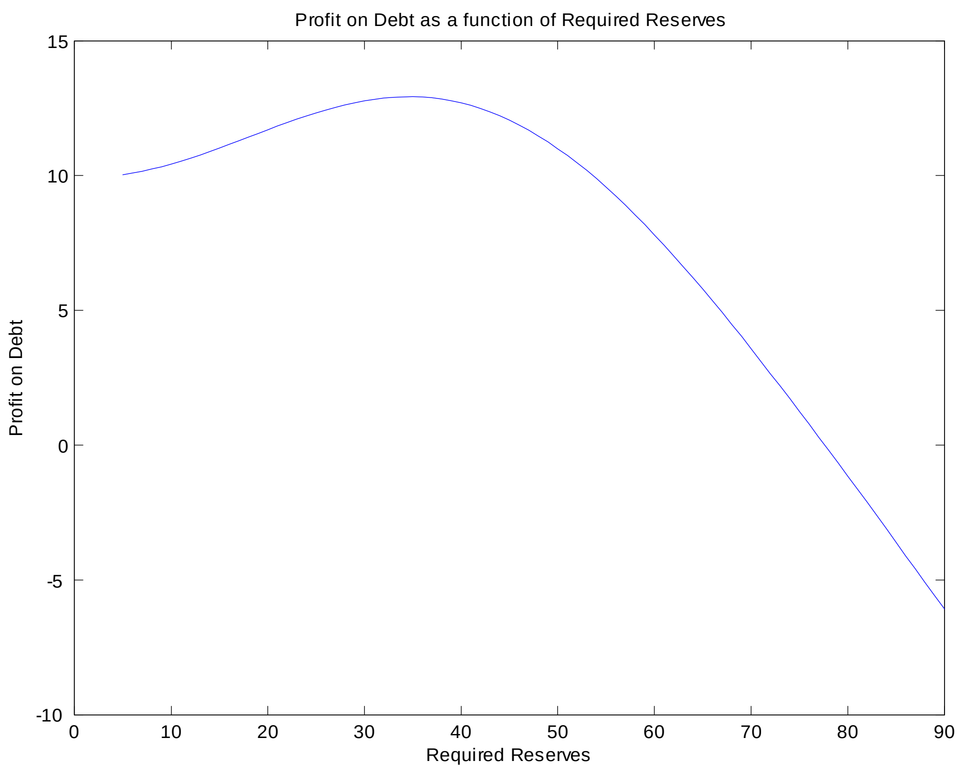

Figure 3: Profit on debt after contributing to reserve capital equally with equity holders. The figure graphs the value of debt as a function of the reserve capital less half the reserve capital as a function of the reserve capital. For low levels of reserve capital there is some incentive to monitor risk by requiring higher reserves but this incentive dissipates for higher levels of reserves.

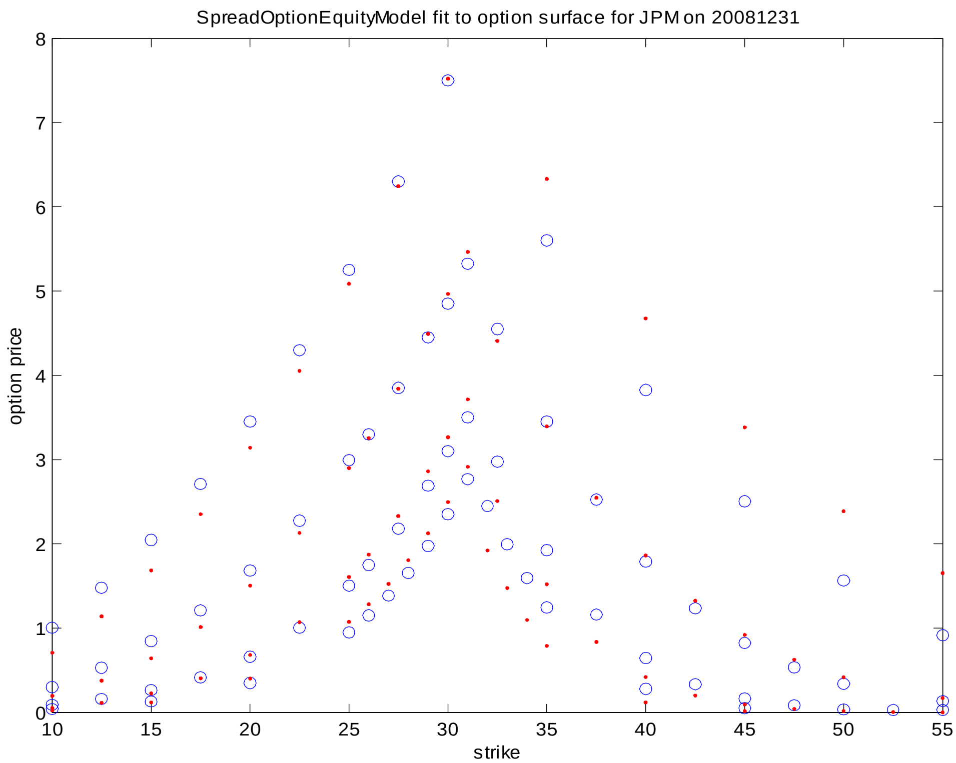

Figure 4: Graph displaying the fit of the compound option model to equity option prices for JPM on December 31 2008. Market Prices are presented as circles while Model Prices are represented by dots.