Dealing with Complex Realities in Financial Modeling

Dilip B Madan — Robert H. Smith School of Business

Ernst Eberlein — University of Freiburg

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We begin our remarks with a general observation. Human beings have to make decisions about uncertain futures over numerous horizons and all such decisions are based on models, be they formal ones or not. A task in this context is to deliver the best set of models supporting decisions that one can formulate recognizing that one may not be able to settle on a single model and we may have to consider a collection of relevant and reasonable possibilities. We will of necessity eventually pick one. We also anticipate that with technological progress and an enhancement in our information processing abilities this collection of models supporting decisions will see improvements and the models will not be static but will evolve in time. Furthermore we expect that different decisions with their focus on different relevant data will employ different models as appropriate ones. Needless to state, there will be errors and we are not guaranteed the choice of the right answer as best we try.

An alternative view is to just use the true model every time, problem solved. Our task is then to focus our energies wholeheartedly on finding this true and trusted model. Many years ago Madan (1983) published a paper entitled, "Inconsistent Theories as Scientific Objectives," by constructing a context in which there were no true models and we cannot presume that one exists in the domain of our description abilities. In any case what does true mean. Models may match reality in some aspects of data generation but this does not mean that the model is the reality. A closer examination can reveal differences. In fact models have their own reality that is distinct from the reality being modeled. The concept of true model is best called into question as possibly the comfort zone for laziness in thought that either hopes for or claims to have the truth, i.e. the one right way to answer all questions. Models are of necessity stylized, simplified and sometimes exaggerated representations of partial realities. Models are therefore not true but nonetheless serve the purposes they are designed to illucidate.

This conflict between models for decisions and models as a record of our understanding of the reality comes to the forefront for those engaged with financial markets. In physics perhaps the latter view is the dominant perspective. In economics, that perhaps occupies a middle ground, there is generally some reliance on time series data and the hope to record some macroeconomic regularities that help us predict the consequences of some actions with a view to formulating economic policy. All this changes in financial market modeling.

Stephen Hawking in his book, "A brief history of time," talks about the day when we would synthesize quantum mechanics and gravity to deliver a unified theory of physics that all will learn and know how the world works. There cannot be such a day for the financial markets as all will try to trade with this model and the very act will destroy the relevance of the model. Models in markets are being pitted against each other in a grand act of mutual destruction. We may take a step back and say that we no longer predict the movement of stocks but just the probabilities, but if we all succeed in this we could make a fortune trading options and even this must be called into question. The markets provide a beautiful play of the power of self referential destruction adequately used by Gödel in proving the incompleteness of number theory. Come not to us with the physics of markets. We are forced to act on beliefs we judge as relevant for the moment, and markets provide a testing ground for competing beliefs that go by the wayside eventually. Many state clearly that they will not invest by taking a view on the probabilities of stock price motion, however estimated. Yet others try to do just that. What are quants to do?

Markets are filled with a multitude of individuals that need to make a host of decisions on numerous questions. For each question and each decision each individual has a view of necessity of what facts in this particular case the contemplated decision can comfortably be made with. The task in question also requires automated delivery with multiple repetitions perhaps with regard to a growing universe of underlying stocks. Translating this conversation into a computer program is the job quants do. They may all do their jobs correctly and yet many will be wrong, requiring new and different facts to be brought into the analysis, but this will be another day and a different model. The new model will be right for a while, but this too will pass, as the conversation develops, computational speeds increase, decision makers change, the facts to be reckoned with are altered and life takes its natural course. A good quant implements equally effectively mutually inconsistent sets of beliefs. Financial modeling is inherently dynamic, wildly exciting, always challenging and never settled.

In this flux of activity surely there must be some stability, some standards, some rules and laws about how things are to be done, what is admissible, what is punishable. Yes, and it lasts for varying lengths of time, some long and some not so long. For example, yield curves were constructed in a fairly fixed manner for many years in the fixed income divisions of banks. This lasted for quite a while and is now giving way to the construction of multiple tenor specific yield curves. There are many agreed upon operating conventions for a variety of activities that last till the next improvement comes along and takes hold of the situation for its lifetime. In a world of continuous improvement there is the way of doing things for the moment, but it is only right for now and not the truth going forward. It can change and yet it may not for quite a while.

Now we are not saying that quants have no role to play in making suggestions on what data to consider, what tests to conduct, what aspects to include in the models and possibly what decisions to take. Quants may well serve the roles of decision makers themselves but the task of modeling is to give shape, structure and context to particular modeling choices that provide us with the ability to simulate, speculate and evaluate as inputs to the final decision making task that we differentiate from the modeling effort.

Let us now take a particular decision that receives considerable attention in practice. It is instructive and helpful to consider this decision as many practitioners, theorists and quants have made confused remarks about how the activity should be conducted, possibly in a manner different from current practice. We shall show that how it is done is exactly how it should be done.

Consider the pricing of a structured investment paying a semiannual 10% coupon for 10 years that pays no coupon if on the coupon date the S&P 500 index is below 700 or the constant maturity 30 year swap rate is below the 2 year constant maturity swap rate. The decision in question are terms at which this security is to be traded. At an abstract level it can be shown (Cherny and Madan (2010)) that the bid and ask prices for the security should be the infimum and supremum over a set of valuation measures that belong to the intersection of test measures defining risk acceptability and the collection of risk neutral measures that reprice a set of hedging assets. Intuitively, financial practice is to name the hedging assets in this case as minimally the surface of options on the S&P 500 index and the constant maturity swap rates along with the discount curve. One then calibrates a variety of models to these instruments and prices the structured investment with an ask near the largest of such valuations and with the bid near the lowest. It is not tolerable to use the physical measure as is evidenced by the demand that you cannot price a complex structure if you cannot price correctly a simple vanilla option with the same methodology. This intuitive demand ties up squarely with what a theoretical analysis of the situation demands. Such practices are also not an exercise in interpolation as the path dependent structure is far more complex than a vanilla option, if anything it is extrapolation, and a fairly complex one at that.

It is also clear in such an activity, both theoretically and in practice, that different products will reach their ask prices with different calibrated models and hence different products will and should be supported by different models. The related support has two dimensions, one describing what partial hedges may be accessible, the second evaluating capital set aside required to cover potential losses related to lack of replication. It is an inconsistency and reveals a lack of understanding of the issues involved to demand that all products be supported by a single model. The single model perspective is appropriate for the myth of complete markets and its associated law of one price, but financial market participants intuitively recognize this myth as such. They are surrounded by incompleteness and instinctively behave in the manner that is theoretically correct for this context. Industrial practice, good sense and theoretical analysis all reach the same conclusion.

What is the test of reality in this exercise? It is not about predicting anything. It is about having a careful discussion about what are the relevant test measures and hedging assets to put into the mix. Additionally one hopes to develop an understanding of what exactly are the risks embedded in the product. Finally one has to determine at best what can be inferred as the prices associated for these risks along with whatever hedging is possible. This is what is done and it is what should be done. Proclamations to the contrary are but an illustration of the confusion resulting most likely from clinging to the text books of what the world would look like if markets were complete. They are not and we behave, sometimes unknowingly, but in any case correctly. Remember, the hedging strategies are not necessarily dynamic trading in the stocks under some unknown, unapproved and generally unknowable physical measure extending out for 10 years, but often static hedging using the option surface to begin with. Dynamic hedging of variance swaps being a good counterexample where there is sufficient confidence in the dynamic filtration to proceed otherwise. How, why, where, when and who gets sufficient confidence in underlying assumptions to act on them is perhaps the most difficult question and certainly beyond what financial expertise we may possess.

In this regard it is instructive to note that with respect to the pricing of CDO tranches that got problematic during and after the financial crisis of 2008, the market was left with little analytical model support. This is because it was known almost from the beginning that attempts based on the normal distribution were incapable of matching market quotes (Eberlein, Frey and von Hammerstein (2008)). In the absence of a formal evaluation of risk exposures and their terms there was plenty of room for an exaggeration of errors based on heuristic and informal valuations. When models fail they are overridden by traders and it has been a recognized problem that valuations based on trader overrides are biased in the trader's favor. This is sometimes dealt with by demanding greater capital requirements commensurate with the level of overrides. We are not aware of the exact situation in this regard with these markets through the crisis. Equally important was the absence of hedging instruments in this domain. Some partial attempts were being made with regard to trading some housing stock price indices.

References

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