Capital requirements, the option surface, market, credit and liquidity risk

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Abstract

The Sato process model for option prices is expanded to accomodate credit considerations by incorporating a single jump to default occuring at an independent random time with a Weibull distribution. Explicit formulas, in this context, for the bid and ask prices of two price economies that price residual risks to levels of risk acceptability are then derived. Liquidity considerations are thereby captured by the movements in the two prices that indirectly reflect changes occuring in the underlying set of zero cost risky cash flows acceptable to the market. In such two price economies it has been proposed that capital requirements supporting a trade are to be set at the difference between the ask and bid prices of the two price economy. We proceed to evaluate the variations in the level of such required capital over time. In particular we observe that the Lehman bankruptcy was primarily a liquidity event for the remaining banks from the perspective of changes in the levels of such a required capital. Additionally, we observe that variations in such capital requirements over time are primarily explained by movements in the option surface and the levels of liquidity, with credit variations playing a part occasionally. The estimations

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conducted in the paper are novel to the literature on option pricing as we estimate for the first time a closed form model for the two price data of bid and ask option prices whereas most of the literature heretofore has estimated a single risk neutral price on data for midquotes. There are therefore no comparative benchmarks in the literature for the modeling conducted here.

1 Introduction

Risk in the market for derivatives has a number of dimensions of interest to those seeking to regulate these markets. In general apart from price movements of the underlying assets, we have the risk of movements in the corresponding option surfaces. These movements capture changes in asset volatility, the volatility of volatility, the skewness of the risk neutral distribution and the term structure of at-the-money volatility. Their relevance arises from the use of options as hedging instruments supporting the pricing and risk management of the more complex structured investments. Additionally there are the risks of changes to market liquidity along with variations in the credit standing of the underlying entity also embedded in the prices of derivatives. The market for credit default swaps provides the hedge for credit while liquidity changes are seen in the secondary market for derivatives directly. From a regulatory standpoint it is critical that capital requirements be set in a risk sensitive manner with a view to counteracting adverse risk incentives inherited by limited liability contracts embedded in the derivatives world of potentially unbounded liabilities, as argued for example in Madan (2009), and Eberlein and Madan (2010). It is therefore imperative that we understand how all these varied risk dimensions impact derivative capital requirements and how in particular they behaved during the crisis, presumably peaking at the date of the Lehman bankruptcy.

Such an evaluation requires an implementable theory of risk sensitive capital requirements that one may apply to a set of stylised trade positions. The most widely used measure of risk in setting capital requiriments is the value at risk (VAR). The relationship of required capital to VAR is imprecise and lacks a theoretical foundation. The VAR measure itself has been criticized as a basis for capital requirements in Artzner, Delbaen Eber and Heath (1999) where refinements or corrections were proposed. A number of operational refinements were then developed in Cherny and Madan (2009) that was followed by the development of the theory of two price economies in Cherny and Madan (2010). The theory of two price economies was then applied to the problem of capital prescriptions by Carr, Madan and Vicente Alvarez (2011) and it is this methodology that we implement and evaluate here. However, we regard this development as essentially a refinement of VAR.

The two prices of a two price economy are termed bid and ask prices but they are not to be confused with the bid and ask prices of relatively liquid markets, like the market for stocks where presumably the law of one price prevails and bid ask spreads reflect the costs of inventory management and/or the asymmetric information costs of market makers. With regard to bid ask spreads in liquid markets we cite Copeland and Galai (1983), Easley and O'Hara (1987), Glosten and Milgrom (1985). Ahimud and Mendelson (1980), Demsetz (1968), Ho and Stoll (1981, 1983) and Stoll (1978) focus particularly on the order processing and inventory costs of liquidity providers. There have also been numerous statistical studies on the bid ask spread (Roll (1984), Choi, Salandro and Shastri (1988), George, Kaul, and Nimalendran (1991), and Stoll (1989)). In particular Huang and Stoll (1997) consider decomposing the spread into order processing, inventory and adverse selection components. These are not the spreads that are modeled in the two price markets of conic finance where the law of one price fails but are instead the spreads associated with the provision of liquidity in highly liquid one price markets.

Yet another approach to spreads in the literature is the introduction of transaction costs (Constantinides (1986), Jouini and Kallal (1995), Lo, Mamaysky and Wang (2004)). The spread now reflects the commission charges of trading and may be related to various empirical aspects of the asset in question including the order flow and the trading volume. These studies also address the costs of trading in relatively liquid markets.

On the other hand there is a large segment of financial markets that creates financial products using the relatively liquid markets for hedging. These are the markets for structured investments or over the counter structured products. Most transacting is infrequent and one generally buys from a provider at the ask price. In case an unwind becomes necessary one sells back to the provider at a substantially reduced bid price. The spreads here are not related to inventory considerations as both parties generally hold the positions out to an explicitly stated contract maturity. In the two price markets of conic finance the focus of attention is not on a spread around a single risk neutral price at which one may in principle trade in both directions, but shifts to modeling directly the two separate prices at which transactions occur. Carr, Madan and Vicente Alvarez (2011) show that the midquote in such two price markets will generally not equal the risk neutral value. Madan and Schoutens (2011) further show that were midquotes taken as candidates for the risk neutral value then there would be static arbitrage opportunities all over the place.

In the equilibrium of two price economies the bid price is now seen as a minimal conservative valuation such that the expected outcome will safely exceed this price under numerous alternative valuation possibilities. Similarly an ask price is a maximal valuation ensuring that the expected payout will fall below the price under a similar set of alternative valuation possibilities. The spreads of conic finance are then tied to the specification of the set of valuation possibilities being entertained. A positive expectation under all the valuation possibilities defines the set of risks acceptable to the market, seen now as a passive counterparty to all financial transactions. Agents are not modeled as trading with each other but just with an abstract market that has no views, preferences or endowments, but merely tests every proposed transaction for acceptability using its set of valuation possibilities. Conic finance provides us with a formal model of the abstract market that differs from the classical market. The latter is associated with the law of one price where all transactions with a positive expectation under the single market pricing kernel are accepted. This is a half space of acceptable risks that is replaced in conic finance by a proper cone containing the nonnegative cash flows.

Given theoretically these two prices we follow Madan and Schoutens (2010), and Carr, Madan and Vicente Alvarez (2010) and define capital reserves for derivative liabilities as the difference between the ask and bid prices. Theoretically a liability could be unwound by buying it back from the market at the ask price and holding reserves at this level would be quite safe. But it would also be quite a substantial amount of capital that allows no use of funds on taking on the liability because a capital reserve set at the ask exceeds any possible price. Madan and Schoutens (2010) and Carr, Madan and Vicente Alvarez (2010) argue for releasing a conservative valuation like the bid price and holding just the difference as a reserve. Assuming that this bid price could be recovered one could couple this with the reserve to cover the unwind at the required ask.

When trust disappears in the market potential transactions have to pass a more stringent collection of tests to be approved. This situation is analytically captured by expanding the set of valuation or test measures under which a positive expectation is being demanded. As a consequence bid prices fall, ask prices rise, and there is a resulting expansion of capital requirements limiting economic activity. The two prices of conic finance attempt to calibrate trust in the market place by explicitly modeling the cone of acceptable risks.

Liquidity risk is then captured by movements in trust as reflected by the cone of risks acceptable to the market. The market is seen as reducing the set of classically acceptable risks defined by a positive risk neutral expectation, by requiring a positive expectation under additional valuation possibilities as well. The existence of these additional valuation possibilities introduces the two prices of conic finance and liquidity issues. The original risk neutral measure does all the pricing of classical risks using the linear pricing rule induced by the risk neutral measure. Liquidity risk pricing is nonlinear as the two prices are seen as infima and suprema of a set of valuations making the measure change attaining the two prices dependent on the cash flow being priced and hence the nonlinearity.

For a candidate classical risk neutral measure for the option surface we synthesize the risks by the four parameter model of the Sato process introduced in Carr, Geman, Madan and Yor (2007), that is based on the variance gamma (VG) law (Madan and Seneta (1990), Madan, Carr and Chang (1998)) at unit time . The Sato process was shown in Carr, Geman, Madan and Yor (2007) to be particularly effective in synthesizing options across numerous strikes and maturities at a point of time by four parameters. The model is a one dimensional Markov model and in the absence of static arbitrage there must exist such a model (Carr and Madan (2005), Davis and Hobson (2007)). Hence we employ it as an adequate summary of the option surface at a point of time.

In addition to the risks of movement in the underlying price and the risk neutral parameters describing the surface of option prices we wish to simultaneously synthesize movements in credit and liquidity risk. In this regard we note that traditionally credit and liquidity have been empirically analysed by looking for securities with the same credit exposure and different liquidities with any remaining price differences being then attributed to the liquidity differences (Ahimud and Mendelson (1991)) or by controlling for liquidity differentials in the estimation of credit exposures (Tibor, Jarrow and Yildrim (2002)). There are few models parameterizing both aspects in the same model that then allows the estimation to sort out the relative impacts. Furthermore liquidity and credit issues have primarily been studied in the market for stocks and bonds. With regard to options Cetin, Jarrow, Protter (2004) and Cetin, Jarrow, Protter and Warachka (2006) consider liquidity costs inherited by options markets when the market for the underlying asset is not liquid. As a result liquidity costs are incurred by the hedge. We also cite in this connection Cetin, Soner and Touzi (2007). For the approach taken here the underlying asset remains liquid but as we do not have the possibility of complete replication, bid and ask price spreads reflect charges for the need to hold residual risk. Additionally the formulation presented here simultaneously addresses both credit and liquidity issues in the market for derivatives with the focus on capital reserves in the place of pricing or valuation.

As Cetin, Jarrow, Protter and Warachka (2006) write,

"*Risk management is concerned with controlling three financial risks: market risk, credit risk and liquidity risk. Starting with the Black Scholes-Merton option pricing formula, both market and credit risk have been successfully modeled with Duffie (1996) and Bielecki and Rutkowski (2002) offering excellent summaries of these literatures. In contrast, our understanding of liquidity risk is still preliminary.* "

We therefore seek to first extend the Sato process model to accomodate credit risk. In this direction there is already a substantial literature and we cite for example Davis and Lischka (2002), Andersen and Buffum (2003), Albanese and Chen (2005), Linetsky (2006), Atlan and Leblanc (2005) and Carr and Madan (2009) that allows in particular for linkages between comovements in the underlying asset price and the probability of the credit event. In this paper that is an initial foray into jointly modeling both credit and liquidity risk we take a first order approach to credit risk by allowing for its mere existence but ignoring issues of comovements that may now exist in principle in all the three dimensions of market, credit and liquidity. Extensions addressing and then modeling aspects of comovement are here left for future research. With regard to both credit and liquidity we merely allow for existence. We therefore employ a simple model for the credit event and use a Weibull distribution for an independent time of default.

As already noted both the study of market and credit risk are substantially advanced and we have much to borrow from, making some particular choices suitable to the context. The study of liquidity risk is relatively preliminary but it is fairly widely acknowledged that these risks are at play when spreads are significant enough to deter for example high frequency trading in the associated assets. The bid and ask spreads in stocks and a variety of fixed income securities that have many market participants employing high frequency trading strategies may well be related to the various market maker considerations modeled in the literature for such spread analysis, but as already noted these are the spreads of the relatively liquid markets. Many financial contracts are traded outside such markets where the two prices are just that, the prices for buying from or selling to the market, and we then need a theory for such two price markets. Option markets are probably in between these extremes with some liquidity but yet with many shorter maturity out of the money positions being held to maturity.

We adopt the theory of two price markets proposed in Cherny and Madan

(2010) and apply it here to the test case of option markets, treating it for the purposes of this paper as a proper two price market, thereby ignoring the little liquidity that it does have. Proper two price markets would include the whole host of structured products that are now an established part of the financial markets. However, the structured product markets lack access to published price data. We are therefore employing option markets as a proxy for the two price markets studied in Cherny and Madan (2010). In such two price markets liquidity risk moves away from the law of one price and the associated linear pricing rule, to a nonlinear pricing rule for liquidity risk. Liquidity risk is therefore fundamentally different from market and credit risk as the latter two fall within the classical domain of a linear pricing rule. This observation may help explain the difficulties associated with modeling liquidity risk as an analysis of liquidity may require a paradigm shift in the approach to pricing, viz. a theory for two price markets.

Combining the three considerations of market, credit and liquidity risk we obtain a model yielding closed forms for the bid and ask prices of two price economies with four parameters that synthesize the option surface. Furthermore, the Weibull distribution provides two credit parameters in the expected life or scale of time to default and the shape parameter yielding the sensitivity of the hazard rate to the firm's age. Market and credit risk are modeled within the classical purview of a linear pricing rule. Finally we introduce two parameters capturing movements in the cone of acceptable risks that may be termed the levels of risk aversion and the absence of gain enticement. These are the liquidity parameters of the model yielding nonlinear pricing models for the bid and ask prices of two price economies. In all there are eight parameters in the full model.

We note in this regard that the model proposed here and its estimation is a novel addition to the literature as we estimate parameters using separately both the bid and ask prices of the option surface. The literature heretofore typically estimates a single risk neutral measure using the midquote as a candidate for the one price of a market satisfying the law of one price. Carr, Madan and Vicente Alvarez (2011) and Madan and Schoutens (2011) have observed that the midquote of our two price economy in fact deviates from the base risk neutral valuation.

We then go on to employ the perspective of two price markets to study the capital requirements proposed in Carr, Madan and Vicente Alvarez (2011). In particular we describe how capital responds to volatility, the movements of the option surface, credit considerations and the newer modeling of movements in the cone of acceptable risks.

The eight parameters are estimated on data for bid and ask prices for options on four financial firms with sufficient data in the selected period. The estimation is conducted every three days for three years beginning October 23, 2007 and ending September 22, 2010. Capital requirements are then assessed for a variety of options each day and we present an analysis of the contributions of the various risk sources to variations in required capital reserves. In practice capital would be set at some level of aggregation. For example a 50 million dollar issue of a structured investment on three underliers with a hedge in place would be analysed for a capital requirement as a package deal. It could on occasion be coupled with other structured investments with similar underliers with capital being assessed on the whole portfolio of such issues. Such aggregation permits one to take account of beneficial netting as and when it occurs. Lacking details on such aggregate packages and relying on the conjecture that the qualititative structure of capital dependence on risk components is not affected by the packaging but just the absolute level of the capital involved, we analyse capital requirements at the level of single options.

Furthermore one could seek a comparison of our capital requirements with alternative procedures for comparative purposes. However, we note that the capital requirements we analyse are but a theoretical refinement of traditional value at risk based methods in any case and thereby constitute a representative of all such procedures. Our interest is in ascertaining the decomposition into market, credit and liquidity risk components of such requirements and we expect that this decomposition is not influenced by the particular form of value at risk or its refinement, that is employed.

The outline of the rest of the paper is as follows. Section 1 presents the modification of the stock price model which is driven by a Sato process to accomodate an exposure to default. Section 2 briefly describes the computation of the bid and ask prices of two price economies for a cone of acceptable risks defined via concave distortions. Section 3 presents some stylized facts about how the various parameters impact capital requirements. Section 4 presents the data and estimation results for three years on four financial firms. In Section 5 we construct the time series of capital required for option positions in the four banks in our study. Section 6 decomposes changes in required capital into the various risk components around the Lehman bankruptcy. Section 7 presents a time series for the total and relative contributions to capital activity of the three broad sources of risk, the option surface, liquidity and credit. Section 8 concludes.

2 Accomodating default in derivative pricing

We begin with a brief review of a successful four parameter model that calibrates well option prices across both strike and maturity. This is the Sato process model first introduced by Carr, Geman, Madan and Yor (2007). It was developed as a generalization of Lévy processes that were known to fit option prices well across strikes but could not simultaneously also fit prices across maturity (Konikov and Madan (2002)). The starting point for the construction of this model is a self decomposable law for the risk neutral distribution of the logarithm of the stock price at unit time that we take to be a year. One may associate with such a law at unit time, both a Lévy process and a Sato process, but the latter fits the option surface while the former does not.

Self decomposable laws were studied by Lévy (1937) and Khintchine (1938) and are defined by the property that a random variable X is self decomposable if for every real c, $0 < c < 1$, there exists a random variable $X^{(c)}$ independent of X such that

$$X \stackrel{(d)}{=} cX + X^{(c)},$$

where the equality is in distribution. Lévy (1937) and Khintchine (1938) showed that the self decomposable laws are the class of limit laws. More exactly these are the laws of limits of sequences of sums of independent random variables appropriately scaled and centered. Hence they share the basic intuition of the Gaussian model as representing the limit of a large number of independent shocks.

The self decomposable laws form a proper subclass of the class of infinitely divisible laws with a special structure to their Lévy measures $k(x)dx$. In particular the function $|x|k(x)$ must be decreasing for $x > 0$ and increasing for $x < 0$. An example of such a law is given by the variance gamma (VG) process at unit time and in this case $|x|k(x)$ has the form $\exp(ax - b|x|)$ for $|a| < b$, and we clearly have the required property.

Sato (1991, 1999) showed that one may associate with such a self decomposable law at unit time a process with independent but inhomogeneous increments by defining the marginal laws of the process at time points t upon scaling the law at unit time. Hence we have that

$$X(t) \stackrel{(d)}{=} t^{\gamma} X, \; t > 0.$$

Sato constructed the precise representation for $X(t)$ as an additive process.

Consider T such that for $t < T$, $X(t)$ has a finite time zero exponential moment. Then define $\omega(t)$ by

$$\exp(-\omega(t)) = E_0 \left[\exp \left(X(t) \right) \right].$$

Carr, Geman, Madan and Yor (2007) defined a positive stock price process $S(t)$ with rate of return equal to $r - q$ for an interest rate r and a dividend yield q by

$$S(t) = S(0) \exp((r - q)t + X(t) + \omega(t)).$$

They showed that this simple normalized exponential of an additive process calibrates option surfaces quite well. It is also known that the Lévy process associated with the random variable X at unit time fails to fit the option surface as it has a too fast paced reduction in skewness and excess kurtosis, when compared to model free estimates of these quantities from market data (Konikov and Madan (2002)).

For a specific model we take for X the variance gamma (VG) law at unit time. The classical representation of the VG is as a scaled Brownian motion $W(t)$ with drift, time changed by a gamma process $g(t; \nu)$ with unit mean rate and variance rate ν. This specification for the VG yields the process,

$$X(t; \sigma, \nu, \theta) = \theta g(t; \nu) + \sigma W(g(t; \nu)).$$

The VG process has three parameters σ, ν, θ. The parameter σ, is the volatility of the scaled Brownian motion, ν, represents volatility of volatility or the variance rate of the gamma time change, and θ the drift of the underlying Brownian motion controls the skewness. The gamma process is an increasing pure jump Lévy process with independent identically distributed increments over regular nonoverlapping intervals of length h that are gamma distributed with density $f_h(g)$ where

$$f_h(g) = \frac{g^{\frac{h}{\nu}-1}e^{-\frac{g}{\nu}}}{\nu^{\frac{h}{\nu}}\Gamma\left(\frac{h}{\nu}\right)},\ g > 0.$$

The Sato process constructed from the variance gamma (VG) law at unit time $X(1)$ has an additional scaling parameter γ. In all we have a four parameter model for the option surface. The parameter γ helps to calibrate the term structure of at the money volatility.

We now extend this model in a simple way to merely allow for the possibility of default modeled by a single jump of the stock price to zero. We recognize that numerous formulations in the literature, cited above, already model the hazard rate for the single jump to default as a decreasing function of the stock price. In the interest of jointly modeling credit and liquidity considerations for the first time we take a simpler model for default. Here the logarithm of the stock price process in the absence of default has an additive Sato specification and we take the hazard rate to be purely deterministic and consistent with a time dependent survival probability given by a Weibull distribution. We employ the Weibull distribution as it is a widely used distribution for life times and default times (Lambrecht, Perraudin and Satchell (1997), Lee and Urrutia (1996)). It allows for both increasing and decreasing hazard rates with respect to age. It was used by Madan, Konikov and Marinescu (2006) to infer risk neutral default time distributions embedded in the prices of credit default swap contracts.

We thereby write the defaultable stock price process as

$$\widetilde{S}(t) = \widetilde{S}(0)\exp((r-q)t + X(t) + \omega(t))\frac{\Delta(t)}{p(t)}$$

where the process $\Delta(t)$ starts at one and makes a single move by a jump down to zero at an independent random time τ. Note that as $p(0) = 1$ we have $\widetilde{S}(0) = S(0)$.The probability that $\Delta(t)$ is one is

$$p(t) = \exp\left(-\left(\frac{t}{c}\right)^a\right)$$

where the parameter c controls the scale or average life and the shape parameter a exceeds unity for hazard rates that increase with age, while $a < 1$ otherwise. For $a < 1$ the function $p(t)$ is convex while for $a > 1$ it starts out concave but is eventually convex. The expected life is $\Gamma(1+\frac{1}{a})c^{\frac{1}{a}}$. Even though actual hazard rates under the statistical or real world probability may decrease with age, their risk neutral counterparts we expect tend to be increasing reflecting a risk neutral conditional probability of surviving longer that falls with age (see for example Madan, Konikov and Marinescu (2006)).

Let $F_t(s)$ be the distribution function of the stock price conditional on no default implied by the distribution of the Sato process at time t. Specifically

$$F_t(s) = P\left(S(t) \leq s\right).$$

The distribution function of the defaultable stock price at time t then is

$$\begin{aligned}
\widetilde{F}_t(s) &= P\left(\widetilde{S}(t) \leq s\right) && (1)\\
&= P\left(Default\ by\ t\right) + P(No\ Default\ by\ t\ and\ S(t) \leq sp(t)\ ) && (2)\\
&= 1 - p(t) + p(t)F_t(sp(t)). && (3)
\end{aligned}$$

We shall see that the bid and ask prices for put and call options are determined completely by the stock price distribution function and we employ equation (3) in these expressions to determine bid and ask prices on call and put options on a defaultable underlier.

We note that credit risk is typically analysed by modeling the probability of default and recovery in default (Lando (2009)). For options the recovery is clear as the call is worthless and the put receives the strike. Credit issues then turn on the probability of default.

At this point we have a six parameter distribution function for the price of a defaultable stock. These are the four option surface parameters $\sigma, \nu, \theta, \gamma$ coupled with the parameters of the Weibull survival function c, a. The pricing is also classical at this point with the specification of a single risk neutral law for the underlying asset.

3 Nonlinear Modeling of Liquidity using the Theory of Two Price Markets

We employ here the principles of two price markets set out in Cherny and Madan (2010). The market is modeled as a passive counterparty and all economic agents may trade with the market, delivering to the market cash flows that are market acceptable. The market accepts at zero cost all nonnegative cash flows, and more generally it accepts a convex cone of cash flows containing the nonnegative cash flows. The theory of two price markets differs from the classical one price theory only by reducing the set of cash flows acceptable to the market at zero cost from the half space of positive alpha trades to a proper convex cone containing the nonnegative cash flows. As already noted there are other ways to model bid and ask prices that focus on the microeconomic concerns of market makers providing the liquidity in highly liquid markets. The theory of two price markets expounded in Cherny and Madan (2010) continues to model the market as a classical passive counterparty with the only change being a dependence of the terms of trade on the trade direction. The dependence is however derived from an exogenous specification for the structure of risks that market participants may deposit in the market at zero cost. They may deposit a cash flow nonnegative to the market, but more generally deposit a convex cone containing such nonnegative cash flows. We believe that an investigation of such a minimal departure from the classical model is worthy of an independent investigation before one personalizes counterparties by bringing in game theoretic considerations into the analysis. The focus on possible losses and how conservative the resulting valuations are, depends on the size of the cone of zero cost acceptable risks that constitutes an important primitive defining the market.

Artzner, Delbaen, Eber and Heath (1999) show that all such cones are defined by a convex set of probability measures $\mathcal{M}$, equivalent to the base probability measure of a fixed probability space, with the property that a particular random variable X defined on this space is acceptable if

$$E^Q[X] \geq 0, \text{ for } Q \in \mathcal{M}.$$

Cherny and Madan (2010) use this structure of a cone of acceptable risks to define an abstract market as one that applies this condition to any zero cost cash flow to test it for market acceptability, in that the market will accept it at zero cost or contract to receive it at no initial cost. The set of test measures $\mathcal{M}$ describe all the valuation measures that must approve the acceptability of a random variable. These measures were refered to as scenario measures in Carr, Geman and Madan (2001). To ensure that the set of acceptable risks is smaller than the classical one given by positive expectation under a single risk neutral measure, the set $\mathcal{M}$ should contain a risk neutral measure.

For an operational definition of such cones Cherny and Madan (2010) consider accepting all random variables X with a distribution $F(x) = P(X \leq x)$, provided

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0,$$

for some fixed concave distribution function Ψ. The set of test measures or scenario measures in this case consists of measure changes $Z(u)$ on the unit interval $0 \leq u \leq 1$, with respect to the uniform density for $U = F(X)$ such that the antiderivative L, for $L' = Z$, is bounded by the distortion, i.e. $L \leq \Psi$. We denote this set of test measures $\mathcal{M}^{(\Psi)}$.

Cherny and Madan (2010) then show that the bid price $b(X)$ for a cash flow X with distribution function F is given by the acceptability of $X - b(X)$ and

$$\begin{aligned} b(X) &= \int_{-\infty}^{\infty} x d\Psi(F(x)) && (4) \\ &= \inf_{Q \in \mathcal{M}^{(\Psi)}} E^Q[X]. && (5) \end{aligned}$$

Similarly the ask price $a(X)$ requires the acceptability of $a(X) - X$ and

$$\begin{aligned} a(X) &= -\int_{-\infty}^{\infty} x d\Psi\left(1 - F(-x)\right) && (6) \\ &= \sup_{Q \in \mathcal{M}^{(\Psi)}} E^Q[X]. && (7) \end{aligned}$$

For the specific cash flows associated with call and put options one obtains on integration by parts specific formulas for the bid and ask prices. The bid and ask prices for calls are denoted $C_b(K,t), C_a(K,t)$ while for puts we write $P_b(K,t), P_a(K,t)$ for a strike K and a maturity t. We then have that

$$
\begin{aligned}
C_b(K,t) &= \int_K^\infty (1-\Psi(\widetilde{F}_t(s)))ds \\
C_a(K,t) &= \int_K^\infty \Psi(1-\widetilde{F}_t(s))ds \\
P_b(K,t) &= \int_0^K (1-\Psi(1-\widetilde{F}_t(s)))ds \\
P_a(K,t) &= \int_0^K \Psi(\widetilde{F}_t(s))ds
\end{aligned}
$$

The specific distortion we employ is *minmaxvar*2 introduced in Madan and Schoutens (2010) which is given by

$$\Psi(u) = 1 - \left(1 - u^{\frac{1}{1+\lambda}}\right)^{1+\eta}, \lambda > 0, \eta > 0.$$

Here λ controls the rate at which the derivative of Ψ goes to infinity at zero and represents the coefficient of loss aversion in the market, while η controls the rate at which the derivative of the distortion goes to zero at unity and represents the degree of the absence of gain enticement. Expectation under a concave distortion is also an expectation under a measure change where the measure change is given by $\Psi'(F(x))$ and depends on the cash flow being valued via the distribution function. Higher values of λ induce a greater upward reweighting of losses as this raises $\Psi'(u)$ for u near zero where we have losses and hence one may associate higher values of λ with more risk aversion. Higher values of η on the other hand lower $\Psi'(u)$ for u near unity where we have gains and this reweights gains downwards and so may be associated with a higher absence of gain enticement.

With these two parameters added on we have an eight parameter model for bid and ask prices with the latter two prices being nonlinear as formally the bid is the infimum of valuations while the ask is a supremum of such valuations as per equations (5) and (7). The measure change employed also depends on the asset being valued via its dependence on the distribution function, as the measure change is $\Psi'(F(x))$.

The parameters λ, η are liquidity parameters for when they are increased the set of acceptable risks is reduced, with bid prices falling and ask prices rising. As a consequence any potential offer to sell at a price above the old market bid must now either take a greater price impact for immediate sale or wait longer for a price recovery. Similarly any potential offer to buy below the old market ask has a greater price impact or waiting time. Liquidity risk is typically seen in such price impact terms (Ericsson and Renault (2006)).

Capital requirements are set by the difference between the ask and bid prices as argued in Carr, Madan and Vicente Alvarez (2010) or Madan and Schoutens

(2010). Basically for a liability to constitute an acceptable risk it must be supported by the ask price viewed as the capital or cost of unwinding the position at possibly unfavorable terms. However one gets credit for the bid price as a possible conservative valuation for the position and only the excess need be held in reserve. Note importantly, that the bid and ask prices here are not those associated with the concerns of market makers providing liquidity to markets that trade the associated asset with some high frequency, but rather these are the two prices of a two price economy evaluating conservatively for the acquisition and sale of infrequently traded risks. The difference between the ask and bid prices can also be seen as aggregating what could be lost as an asset with a valuation down to the conservative bid plus what could be lost as a liability with the need to unwind at an unfavorable ask price. The capital reserve is therefore being set with a view towards measuring the possible loss in the contract.

Furthermore from the nonlinear structure of the associated pricing rules of equations (5) and (7) respectively it is clear that a packaged risk has a higher bid and a lower ask than the sum of its components. Hence such price computations should be and would be done at a suitable level of risk aggregation. Most structured products are issued at some level of aggregation in both structure and size of issue. We merely illustrate our computations at the level of data for bid and ask prices of calls and puts. The principles and procedures would in practice be applied at a suitable level of aggregation permitting some netting implicit in the pricing equations. We expect, as already noted in the introduction, that portfolio level capital requirements would be reduced by netting but their qualitative decomposition into contributions from market, credit and liquidity risk would be reflected by the analysis of individual products like options, reported on here.

Some examples of capital requirements set by the spread of ask to bid prices for two price economies help illustrate the procedures. Consider in this regard capital requirements for equity and bond exposures in the classical context of an underlying asset that follows a geometric Brownian motion model with a risk neutral drift equal to the interest rate r and a volatility of σ. The underlying asset value for maturity T with an initial value of 100 is then

$$A(T) = 100 \exp(rT + \sigma\sqrt{T}Z - \sigma^2 T/2)$$

where Z is a standard normal variate. For a pure discount debt with face value F and maturity T the value of debt at time $t < T$ is

$$D(t) = e^{-r(T-t)} E_t[\min(A(T), F)]$$

while the value of equity is

$$J(t) = e^{-r(T-t)} E_t[(A(T) - F)^+]$$

For $T = 10$, $t = 5$, $\sigma = .25$ and a constant interest rate of $r = 5\%$ we sample $A(t)$ on $10,000$ paths for each of which we sample $10,000$ further paths of $A(T)$ to obtain $10,000$ readings for $D_i(5)$, $J_i(5)$ for $i = 1, \cdots, 10,000$. We then determine bid and ask prices using distorted expectations for the distortion *minmaxvar* at a relatively low stress level of 0.1 to get the following results for the bid and ask prices on debt and equity. The bid and ask prices on debt are respectively 35.75 and 38.65. The corresponding values for equity are 19.17 and 28.25. The capital charge for debt is 2.90 while for equity it is 9.08. The greater riskiness of equity is reflected in the Carr, Madan and Vicente Alvarez (2011) capital charges of two price economies.

Given this formulation for risk sensitive capital reserves, the level of reserves is then responsive to movements in the option surface parameters $\sigma, \nu, \theta, \gamma$, the credit parameters c, a, as well as the liquidity parameters λ, η. We shall study the relationship between these parameters and capital requirements first in a stylized setting in the next section and then over a three year data period ending September 22, 2010 for the four financial firms Bank of America BAC, Goldman Sachs GS, J.P.Morgan Chase JPM, and Wells Fargo WFC.

4 Capital sensitivity to parameters in a stylized setting

We take as a base setting for the option surface parameters, the mean value of the estimated parameters across time for the four banks, studied later in the paper. For the liquidity and credit parameters we take a stylized value reflecting a symmetric cone with $\lambda = \eta = .1$. The expected life parameter is set at 5 years and the Weibull shape parameter or hazard rate sensitivity is set at 1.25. These are risk neutral parameter values, and CDS prices are typically quoted most actively at five years, though estimates in section 4 later are larger than five years. The use of a five year life is thereby on the high side. The CDS quote for a five year contract with 60% recovery would on this setting be 750 basis points, a high value. So we are here considering an entity in some financial trouble. Risk neutral hazard rates using a Weibull density were reported well above 1.25 for example in Madan, Konikov and Marinescu (2006). Such a value is consistent with our expectation that risk neutrally surviving longer gets harder with age, even if under the real world measure it may be getting easier. The estimates cited in Madan, Konikov and Marinescu (2006) reflect this expectation.

The estimated parameters in section 4 differ from our base setting here, but the former are calibrated to market bid ask prices for a market that is a stand in for what may be relevant for structured products in general and for capital requirements in particular. We anticipate that regulatory cones of acceptability for relatively infrequently traded products would in general be more conservative than those reflected in market option prices.

The base parameter setting is

σ	ν	θ	γ	λ	η	c	a
0.3725	0.6925	−0.3863	0.4724	0.1	0.1	5	1.25

For a portfolio of options we take 10 options with five strikes and two maturities. The maturities are 3 and 6 months. With the spot level set at 100, and zero interest and dividend yield, the strikes are 80, 90, 100, 110 and 120. The options are out of the money, except the option with a strike of 100 is a call . We first report on the gradient of total capital defined as the sum over all options of the difference between the ask and bid prices for these options. The gradient is computed at the base point with respect to each of the eight parameters. This gradient is given by

σ	ν	θ	γ	λ	η	c	a
74.5244	1.3297	−24.2293	−36.6515	205.6706	183.2942	−2.2387	−22.6903

We observe that volatility, σ, and the volatility of volatility, ν, raise capital requirements while an increase in skewness, θ, improves the return distribution and reduces capital. An increase in the volatility spread, γ, or the scaling parameter lowers capital requirements as it reduces volatilities at each maturity below unity, raising them for the longer maturities. Reducing the cone by raising either the coefficient of loss aversion λ, or raising the coefficient for the absence of gain enticement, η, raises capital requirements. On the credit side we see that lowering the expected life raises capital requirements while an increase in the Weibull shape parameter lowers capital requirements as it raises the growth rate of the stock. The actual effect on capital requirements depends critically not just on the gradient but also on the actual change in the parameters.

In order to better appreciate the difference of the effects of changes in liquidity and credit on bid and ask prices we present a graph of the response of bid and ask prices on a 20% out of the money put and call option for an annual maturity of changes in $\lambda = \eta$ and changes in c for a fixed value of $a = 1.25$. We vary c from 1 to 10 years and vary λ from .05 to .2. The other parameters are as in the base case. We present in Figure 1 the effect of the expected life parameter on bid and ask prices.

Figure 2 shows the effects on the same options of varying the liquidity parameter.

We clearly see the different effects of variations in credit and liquidity on bid and ask prices of options. While for the former both prices move in the same direction the opposite is the case with respect to variations in liquidity. Hence, credit and liquidity are differentiated economic events.

5 Data and calibration summary

The purpose of the empirical analysis is not to test the proposed model. The adequacy of these models for synthesizing option data has been demonstrated in earlier studies and we cite Carr, Geman, Madan and Yor (2007), Cherny and Madan (2010), Carr and Madan (2009) as examples. There are other models that could be used for this purpose like a jump diffusion model or a Lévy process more generally, but as noted in Carr, Geman, Madan and Yor (2007), Lévy processes do not fit the surface of option prices and it was this failure on the part of Lévy processes that led to the development of the Sato process in the first place. Stochastic volatility models could be used to synthesize the surface

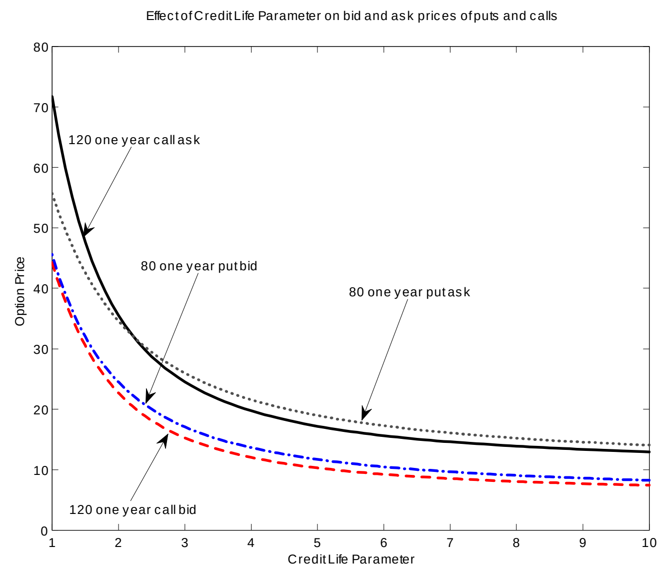

Figure 1: Bid and ask prices for a one year 80 put and a one year 120 call as we vary the expected life parameter

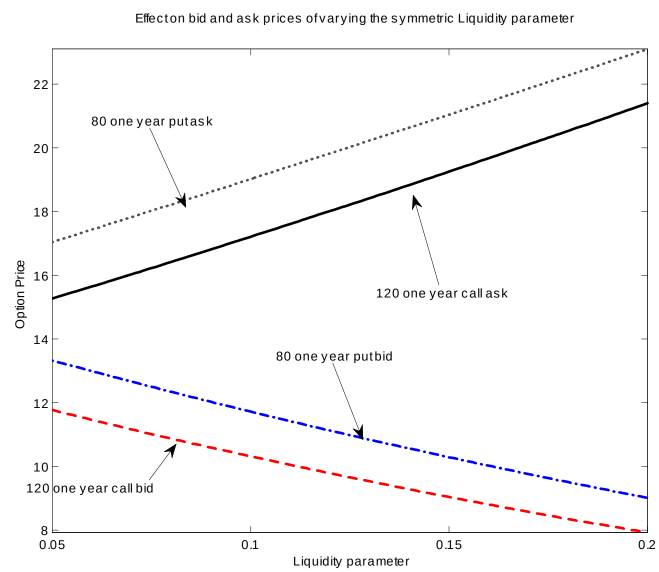

Figure 2: Effect on one year 80 put and one year 120 call of varying the symmetric liquidity parameter.

but they are at a minimum two dimensional Markov processes and it is well known that option prices at a single time point provide little information on mean reversion and the volatility of volatility. Furthermore as shown in Carr and Madan (2005) and Davis and Hobson (2007), the absence of static arbitrage implies the existence of a one dimensional Markov process synthesizing an option surface at a point of time. The Sato process and its enhancement here is such a process. There is also no issue of in sample or out of sample analysis as the estimation is not conducted over any sample period that constitutes the in sample period. The estimation is on the cross section of prices by strike and maturity on a single day.

The object here is to use a mix of established models synthesizing option surfaces at a point of time to estimate a risk neutral law jointly incorporating for the first time market, credit and liquidity components. The risk neutral law is for a point of time and uses data on option prices at one time point only to evaluate the relative contributions to capital attributable to market, credit and liquidity considerations as embedded in the parameters related to these effects. The measure of capital employed is the difference between the two extreme prices of conic finance as a conservative assessment of loss exposure related to unfavorable unwinds.

The eight parameters of our model are calibrated every third day on bid and ask option prices for three years beginning October 23 2007 and ending in September 22, 2010 for the four banks, BAC, GS, JPM, and WFC. Tables 1 to 8 present a summary of the data used. For each of the four stocks we report quarterly averages of 19 variables for 12 quarters covering the three years. The first variable is the average stock price, followed by the average of the first three maturities and the average of the remaining maturities. We then report the average interest rate and dividend yield for the shorter and longer maturity. This is followed by the average strike below and above the spot for the shorter and the longer maturity. Finally we report average bid prices below and above the spot for the first and second maturity spectrums followed by the average ask prices. There are eight tables as we split for each stock the twelve quarters into two sets of six quarters.

There are in all 237 calibrations for each of the four names. Summary statistics for the eight parameter estimates and the corresponding goodness of fit metrics are presented. The goodness of fit metrics are the root mean square error $rmse$, the average absolute error aae, and the average percentage error defined as the average absolute error relative to the average option price in the sample. Also presented are the average number of options used in the calibrations. There are four tables, one for each bank, partitioned into two pieces, one for the parameters and the other for the goodness of fit metrics. Shown are the means, standard deviations and a variety of quantiles for the smoothed parameters and the mean and standard deviations of the goodness of fit statistics.

There were on average 30 to 80 options in the various calibrations. The average percentage error was around 3%. This compares favorably with published and practical experience on such calibrations. Tables 9 through 12 provide the details for the four banks. The calibrations were done in Matlab using the routine $fminunc$ for unconstrained minimization from version 6.5 of Matlab. They took about an hour for each bank or 237 calibrations. We address the stability and robustness later in this section.

We summarize that for BAC the value for σ ranged from .3 to .67 at the 10 and 90 percentile points. The corresponding figures for ν, were .3328, and .9764, for θ, $-0.83, -0.0396$, and for γ, $.3572, .5660$. On the same quantiles loss aversion ranged from 28 to 190 basis points while the absence of gain enticement went from 14 basis points to 84 basis points. The credit parameter related to the average life went from 13 to 41 years, while a ranged from .49 to 4.85.

The comparable statistics for GS were σ, $.3153, .3865$, ν, .4018, 1.0111, $\theta, -.5646$, $-.2148$, and γ, $.3826, .5307$. For loss aversion we have 2 to 155 basis points and gain enticement goes from 20 to 125 basis points. Credit life ranges from 19 to 53 years while a goes from .7349 to 7.6455.

For JPM these values are σ, $.2905, .4488$, ν, .4705, .8939, $\theta, -.6885$, $-.2573$, and γ, $.4020, .5379$. For loss aversion we have 3 to 125 basis points and gain enticement goes from 51 to 161 basis points. Credit life ranges from 13 to 37 years while a goes from .9873 to 4.8703.

Finally for WFC we get σ, $.2161, .4731$, ν, .3950, 1.0302, $\theta, -.8554$, $-.1614$, and γ, $.3609, .5462$. For loss aversion we have 19 to 217 basis points and gain enticement goes from 69 to 267 basis points. Credit life ranges from 12 to 51 years while a goes from .6205 to 4.1181.

Additionally we present in Figures 3 and 4 two graphs of the time series for all the eight parameters. The first graph covers the four option surface parameters for all four banks while the second graph covers the liquidity and credit parameters.

With regard to the robustness, stability and identification of the parameters we report in Tables 13 to 20 the average absolute value of the derivative of the least squares objective function with respect to all eight parameters separately for bid and ask prices for each of 12 quarters. When the absolute sensitivity of the criterion with respect to a parameter is substantial the parameter is typically well identified and the minimization algorithm is not sensitive to local perturbations of the starting value. We observe from these tables that the option surface parameters are well identified. The liquidity parameters reflecting movements in the cone of acceptable risks are also well identified. On the credit side there is a potential lack of identification of the Weibull scale parameter related to the average life, with occasional identification of the shape parameter. We also conducted a similar analysis based on eigenvector decompositions of the derivative of the vector of squared errors with respect to the parameters to reach the same conclusion.

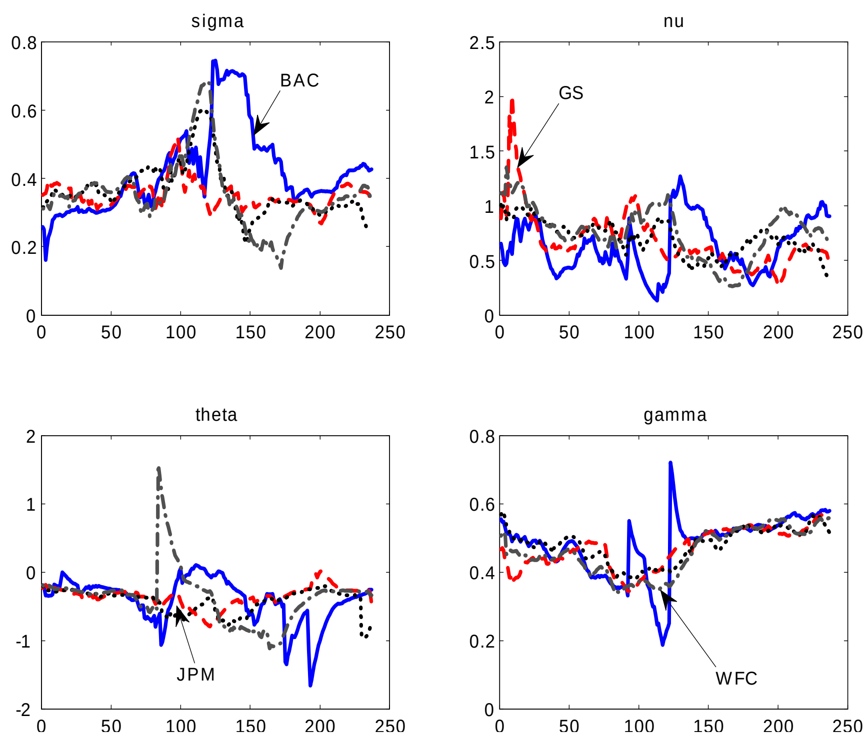

Figure 3: Time series of the four Sato process parameters of volatility σ, volatility of volatility ν, skewness θ and the volatility term structure γ for the four banks. BAC is shown with a solid line, GS with a dashed line, JPM with a dotted dotted line and WFC with a dash dot line.

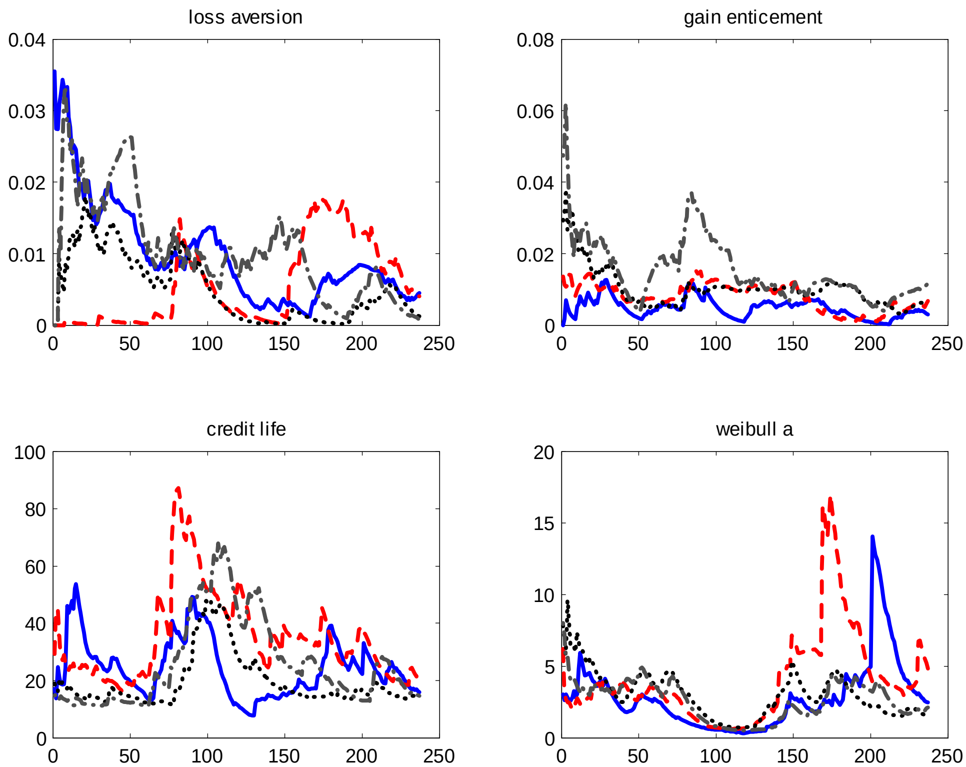

Figure 4: Time series of credit and liquidity parameters, c, a and λ, η for the four banks. BAC is shown with a solid line, GS with a dashed line, JPM with a dotted dotted line and WFC with a dash dot line.

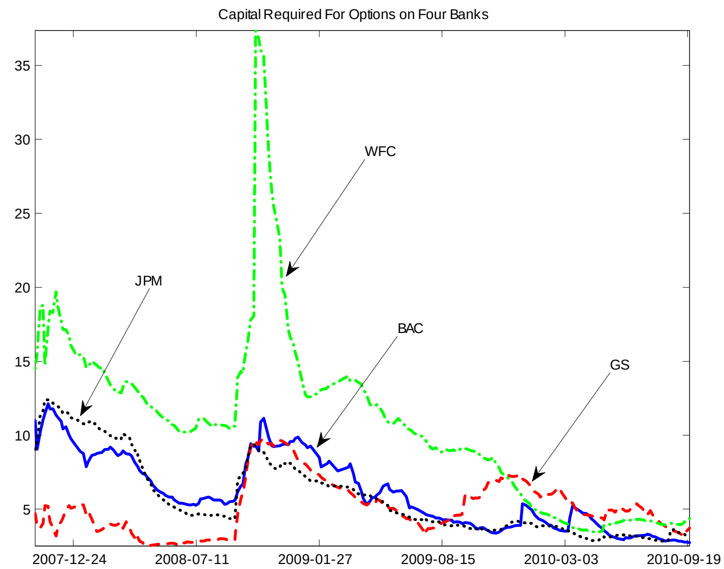

Figure 5: Graph of total capital required on 10 options, 3 calls, 2 puts for the two maturities of 3 and 6 months across time for the four banks, BAC, GS, JPM and WFC.

6 Time Series of Required Capital and their Risk Sensitivities

In this section we construct the capital required on a hypothetical options book of 10 options consisting of five strikes at each of two maturities. We work with a zero interest rate and dividend yield and the maturities are 3 and 6 months with the spot at 100 and strikes at $80, 90, 100, 110, 120$. The options are out of the money, with the exception of the one with the 100 strike which is a call. For each option on each day in our time series we evaluate using smoothed values for the eight parameters of our model the bid and ask prices for each option and the capital required as the difference between the ask and the bid. We then sum the capital required over the 10 options. This is an upper limit of capital for one could have computed the distorted expectation on the portfolio and there would be some advantage to the portfolio. We note in this regard that the three calls are comonotone as are the two puts and hence there is no portfolio advantage within the puts and calls.

We present in Figure 5 a graph of the capital required on the 10 options through our time period. The capital required begins to peak in the fall of 2008 at the peak of the crisis and has fallen steadily from then on.

We also present for each bank the capital required per option separately

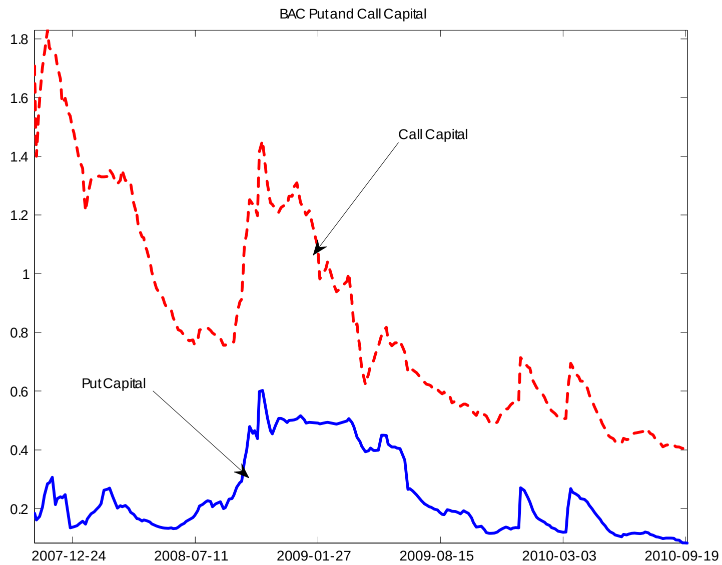

Figure 6: Capital required per call and put separately for BAC across time.

for calls and puts across time. These are presented in Figures 6, 7, 8 and 9 respectively. The call capital is higher than the put capital and initially we thought this was essentially due to the inclusion of the at-the-money call, however the exclusion of the at-the-money call gave the same results.

To understand the effects of strike and maturity we also present the capital required for all options aggregated over all four banks for the 80, 90 put and the 110, 120 call in Figure 10. For the effect of maturity we sum the capital over all options separately for the two maturities and present the graph in Figure 11 .

With a view to understanding the sensitivity of capital to risk components we regressed the required capital for each stock on the eight risk parameters $\sigma, \nu, \theta, \gamma, \lambda, \eta, c$ and a. The results are presented in TABLE 21. We observe that one may explain the capital as a linear function of the risk parameters over this period. The most significant contributors to capital movements are the liquidity parameters followed by the option surface and credit.

7 Capital requirement movements around the Lehman bankruptcy

In this section we enquire into the nature of the Lehman bankruptcy event for the four banks. For this purpose we set up a hypothetical options book of 10 options consisting of five strikes at each of two maturities. We work with a zero

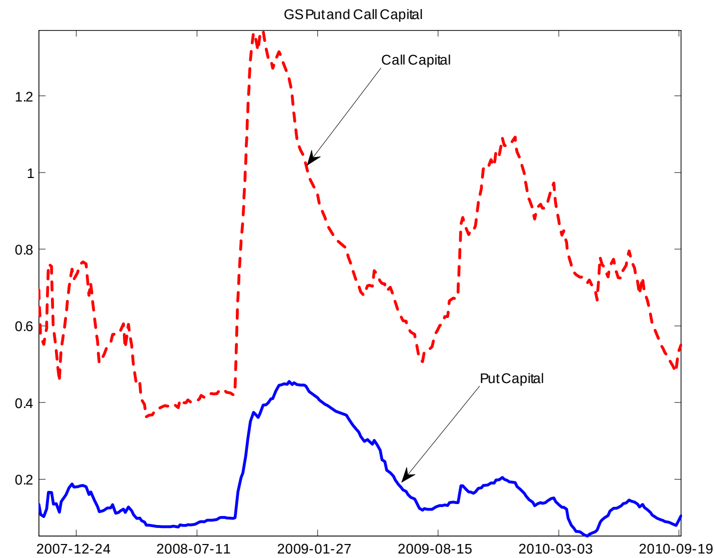

Figure 7: Capital required per call and put separately for GS across time.

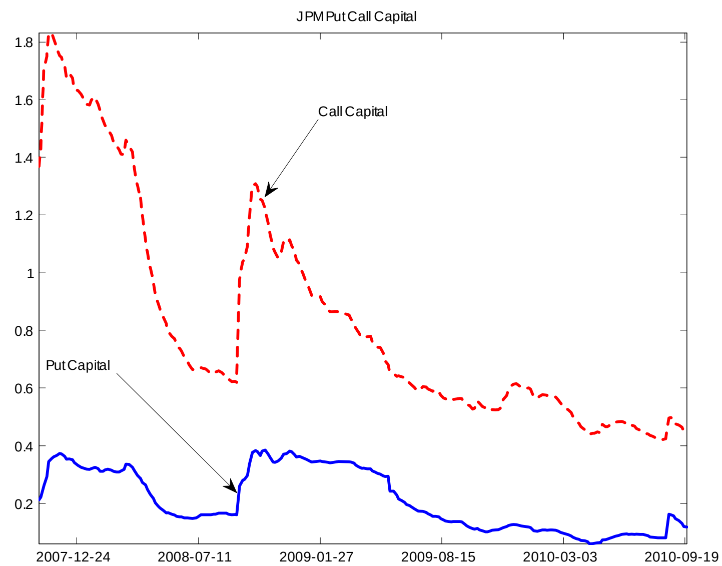

Figure 8: Capital required per call and put separately for JPM across time.

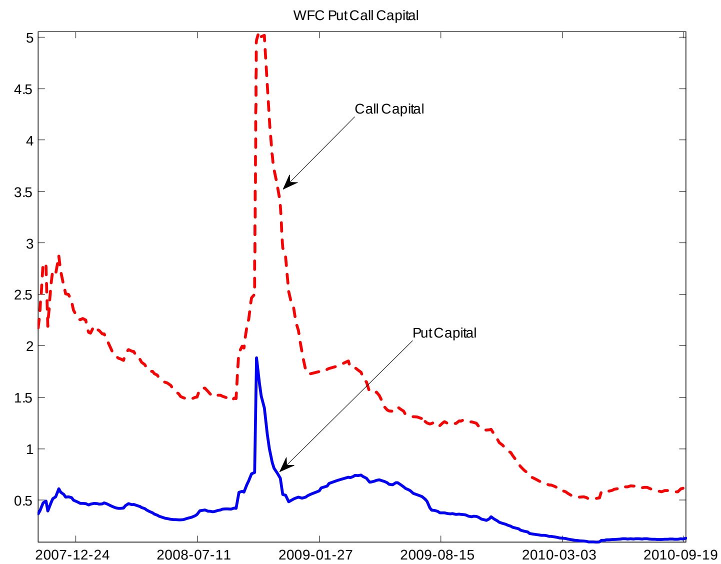

Figure 9: Capital required per call and put separately for WFC across time.

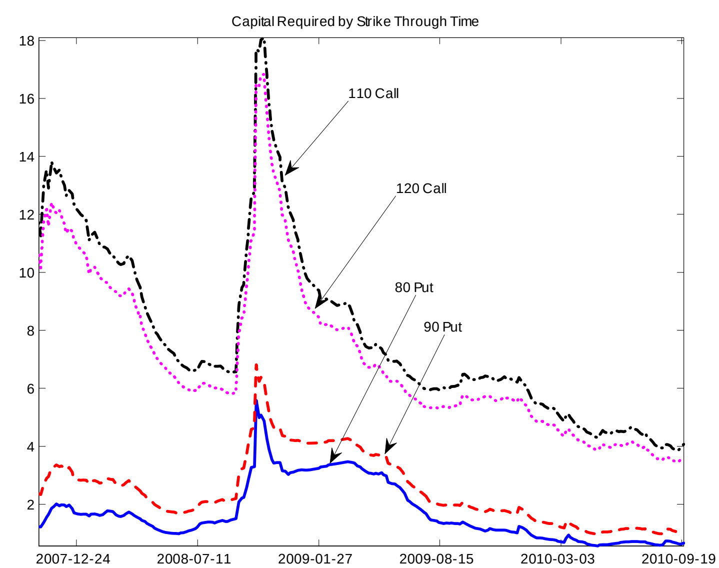

Figure 10: Graph of total capital across time for all four banks and two maturities separately for the out-of-the-money puts and calls.

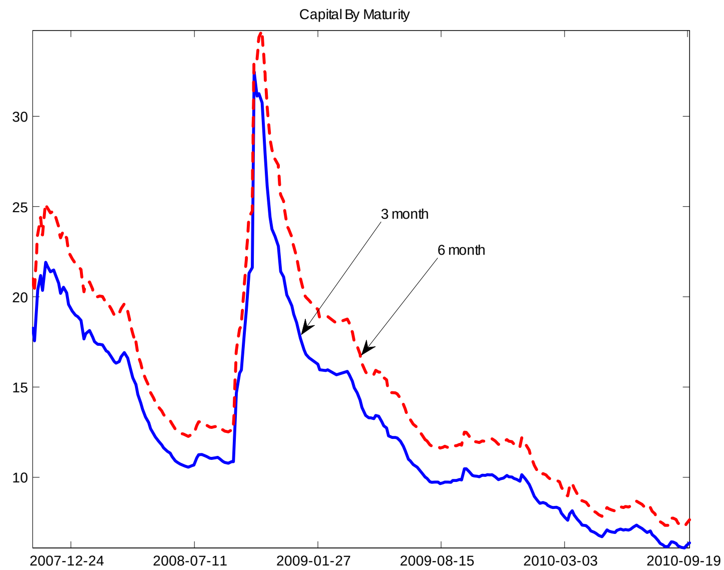

Figure 11: Graph of total capital across time for all four banks and five strikes separately for the two maturities.

interest rate and dividend yield and the maturities are 3 and 6 months with the spot at 100 and strikes at $80, 90, 100, 110, 120$. The options are out of the money, with the exception of the one with the 100 strike which is a call.

For each of the four banks we consider the calibrated parameters about 3 weeks before the bankruptcy at August 26 2008 and three weeks after the bankruptcy at October 8 2008. For each of the ten options we determine using calibrated parameters the bid and ask prices for these options and the capital required measured as the sum over all 10 options of the difference between the ask and bid prices. This total required capital is computed at the two dates for the four banks and the values are displayed in Table 22 along with the percentage increase.

TABLE 22
Pre and post Lehman capital needs
on the hypothetical portfolio of 10 options consisting of
5 strikes of 80, 90, 100, 110 and 120 for a spot of
100 and two maturities of 3 and 6 months. The interest
rate and dividend yield was set at zero for these
calculations. Displayed are the difference between
ask and bid prices using minmaxvar at calibrated
stress levels that distort a calibrated VG based Sato
process for the risk neutral law enhanced with credit
exposure modeled by a Weibull density for the default
time

	BAC	GS	JPM	WFC
Pre Lehman	2.3684	1.1851	2.0325	4.5648
Post Lehman	5.2694	3.8898	4.4995	8.3947
Percentage increase	122.48	228.22	121.38	83.89

These are significant increases in capital requirements at market calibrated stress levels for the cones of acceptable risks. Regulatory settings could even be more conservative than these values.

We now decompose this increase in capital requirements into eight risk sources represented by changes in the eight parameters. The capital increases are computed for hypothetical options on returns with the spot at 100. Any changes in the capital requirements are then due to variations in the parameters. Since capital is now seen as a deterministic function of the form $c = g(\Theta)$ one may approximate the change by

$$\Delta c \approx \left(\frac{\partial g}{\partial \Theta} |_{\Theta_0} \right) \Delta\Theta. \tag{8}$$

We compute the gradient vector at the parameter point for August 26, 2008 and then evaluate the product of the gradient with the change in the parameter value between October 8, 2008 and August 26, 2008. We determine the contribution of each parameter as given by the product of the gradient with respect to the parameter times the change in the parameter. The relative contribution is then obtained on dividing the contribution by the right hand side of equation (8).

The relative contributions are given in Table 6 for the four banks.

TABLE 23
Relative parameter contributions to capital requirements from pre to post Lehman bankruptcy. Displayed are first order estimates of change in capital computed by the gradient of capital with respect to the parameter on August 26 2008 times the change in the parameter between October 8 2008 and August 26 2008 for the four banks and eight parameters.

	BAC	GS	JPM	WFC
σ	0.0406	-0.0657	0.0136	0.1003
ν	0.0254	-0.0026	0.0002	0.0192
θ	0.3476	0.0526	0.0307	-0.0264
γ	0.0409	0.0672	0.0750	0.1077
λ	0.0374	0.8513	0.3998	0.0673
η	0.4854	0.0972	0.4808	0.7318
c	-0.0073	0.0	0.0	0.0
a	0.0299	0.0	0.0	0.0

We see from Table 23 that the major contribution to changes in capital requirements came in this instance from movements in the liquidity parameters. Changes in credit played a minor role. On this evidence it is suggested that the Lehman bankruptcy was primarily a liquidity event and not a credit event for the other large banks.

8 Capital activity and risk contributions across time

We report here on the total and relative contribution to changes in capital requirements of parameter movements. For this purpose we employ the smoothed parameter values at each calibration date. The change in capital attributed to a parameter is measured by the absolute value of the sum over all ten options of the gradient of capital required for the option times the change in the parameter to the next calibration date. The gradient is computed at the calibration date. For the relative contribution we divide these positive parametric contributions by their sum.

We then partition the relative changes into three groups: The option surface, liquidity and credit. The option surface contribution is given by the sum over the contributions of $\sigma, \nu, \theta, \gamma$. For liquidity we sum the contributions of λ, η while for credit we sum the contributions of c, a. Figures 12, 13, 14 and 15 show the total and relative contributions of the three sets of risks on capital activity for the four banks through the three years ending September 22, 2010.

From these figures it is obvious that liquidity and the option surface are the major contributors to variations in required capital with credit occasionally playing a part.

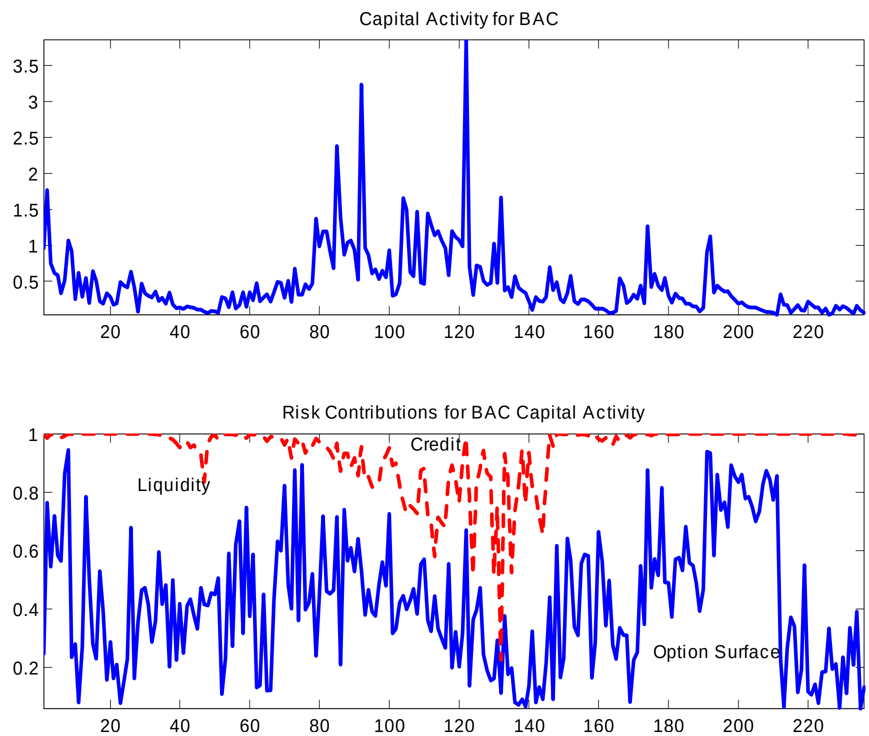

Figure 12: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for BAC.

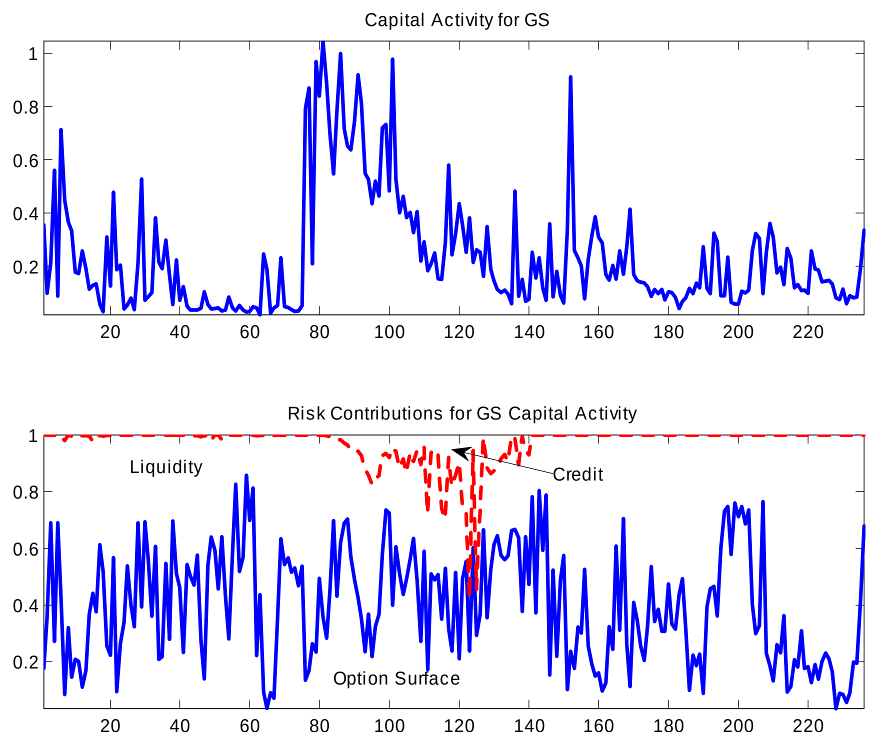

Figure 13: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for GS.

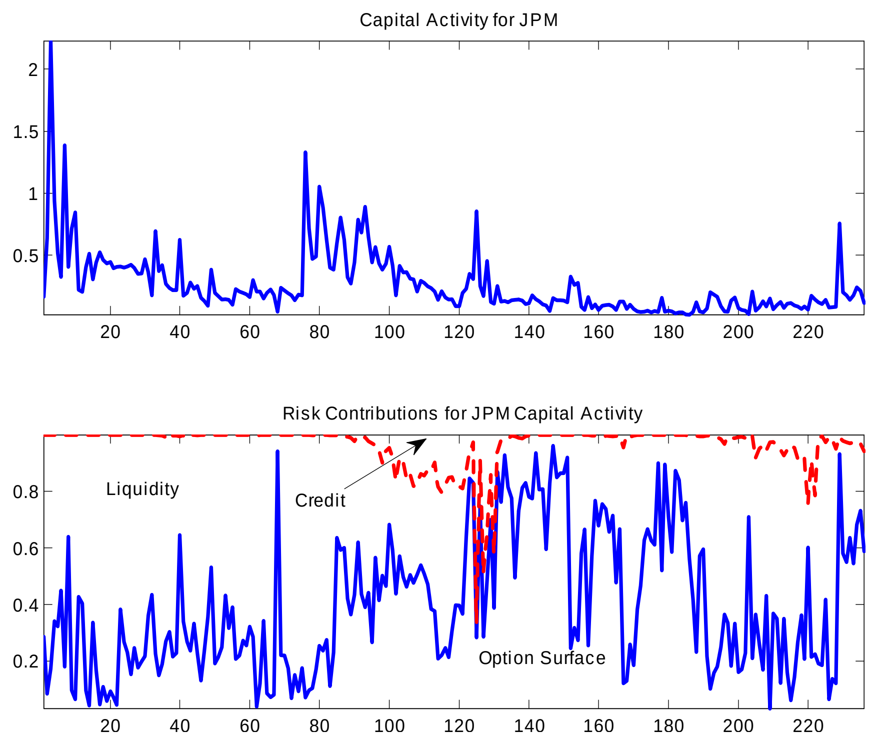

Figure 14: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for JPM.

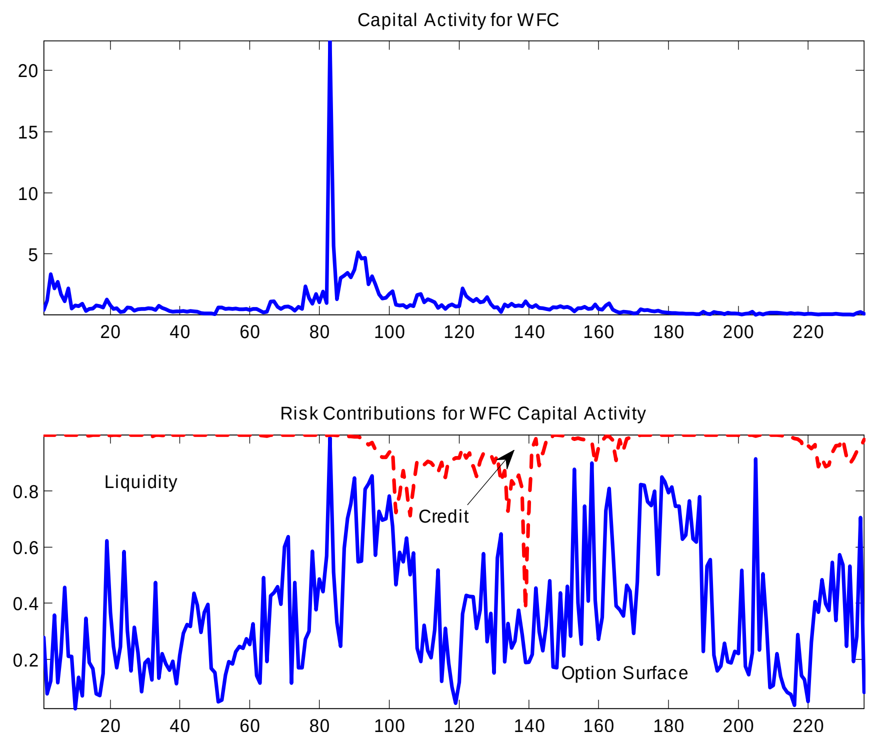

Figure 15: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for WFC.

9 Conclusion

The Sato process model for option prices introduced in Carr, Geman, Madan and Yor (2007) is expanded to accomodate credit considerations by incorporating a single jump to default occuring at an independent random time with a Weibull distribution. Following Cherny and Madan (2010) explicit formulas for bid and ask prices of two price economies are constructed. Movements in these prices are then seen as synthesizing the effects on required capital of changes to the option surface, credit and liquidity. The final model for bid and ask prices of put and call options has eight parameters, four from the Sato process, two from the credit market and the underlying Weibull distribution and two for liquidity.

We observe additionally that the modeling and estimation conducted here constitutes a novel addition to the literature as we estimate for the first time (excepting Cherny and Madan (2010)) a model for both the bid and ask prices of the option surface. The literature has heretofore estimated a single risk neutral model using the midquote as a candidate for the one price of a market satisfying the law of one price. Madan and Schoutens (2011) observe that the midquote of our two price economy in fact deviates from a risk neutral valuation. There are therefore no benchmarks for the exercise conducted here, in the extant literature.

This eight parameter model is estimated on daily option pricing data for four banks over a three year period ending September 22, 2010. We follow Carr, Madan and Vicente Alvarez (2010) and define capital requirements supporting a trade by the difference between the ask and bid prices. From the perspective of variations in capital required measured with respect to a hypothetical portfolio for options on returns it is observed that the Lehman bankruptcy was primarily a liquidity event for the other banks. We also observe that for explaining the variation in capital requirements over time, a major role is played by the option surface and liquidity considerations with credit playing a part occasionally.

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TABLE 1

BAC

	Q1	Q2	Q3	Q4	Q5	Q6
Spot Price	39.2447	40.1652	31.7742	30.5445	18.7692	5.9502
Avg. First Maturity	0.2218	0.2744	0.2574	0.2437	0.2004	0.1929
Avg. Second Maturity	1.1496	1.2337	1.0200	0.8970	1.1406	0.9188
Avg. First Interest Rate	0.0421	0.0297	0.0254	0.0279	0.0222	0.0044
Avg. Second Interest Rate	0.0377	0.0264	0.0285	0.0302	0.0219	0.0069
Avg. First Div. Yld.	0.0489	0.0528	0.0598	0.0753	0.0570	0.0085
Avg. Second Div. Yld.	0.0523	0.0631	0.0590	0.0594	0.0472	0.0131
Avg. First Strike below Spot	34.1791	34.1652	26.8764	26.0625	15.9729	5.1287
Avg. First Strike above Spot	44.2476	45.6459	36.3498	35.1276	21.4063	6.8338
Avg. Second Strike below Spot	33.9958	34.3497	26.5813	25.9026	15.9357	5.1350
Avg. Second Strike above Spot	44.7407	46.0845	36.3008	35.3312	21.4155	7.0250
Avg. First Bid Price below Spot	1.1241	1.4156	1.0730	2.1195	1.9813	1.0003
Avg. Second Bid Price below Spot	3.5359	4.4784	2.8898	4.1296	4.0684	1.8759
Avg. First Bid Price above Spot	0.7901	1.0965	0.9079	1.6362	1.8893	0.9948
Avg. Second Bid Price above Spot	2.6818	2.9396	2.4584	3.2888	4.0526	1.9279
Avg. First Ask Price below Spot	1.2085	1.4973	1.1056	2.2159	2.0421	1.0210
Avg. Second Ask Price below Spot	3.8154	4.7905	2.9995	4.3312	4.2493	1.9224
Avg. First Ask Price above Spot	0.8694	1.1837	0.9399	1.7222	1.9583	1.0218
Avg. Second Ask Price above Spot	2.9502	3.2008	2.5570	3.4826	4.2570	1.9993

TABLE 2

BAC

	Q7	Q8	Q9	Q10	Q11	Q12
Spot Price	10.3948	14.7307	16.4428	15.6618	17.4109	14.3793
Avg. First Maturity	0.2820	0.2430	0.2100	0.2772	0.2803	0.2572
Avg. Second Maturity	1.1013	0.7824	1.0305	1.3788	1.1544	0.8333
Avg. First Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Strike below Spot	8.8842	12.6246	14.0609	13.3499	14.8413	12.1789
Avg. First Strike above Spot	11.9819	17.0438	18.9473	17.9254	19.7929	16.5479
Avg. Second Strike below Spot	8.8850	12.5965	14.1901	13.5347	14.8144	12.2617
Avg. Second Strike above Spot	11.5029	16.9671	19.0604	18.1304	20.1352	16.6749
Avg. First Bid Price below Spot	1.3425	0.8469	0.6382	0.5282	0.6041	0.5076
Avg. Second Bid Price below Spot	2.6667	1.8795	2.0316	1.8500	1.7617	1.3667
Avg. First Bid Price above Spot	1.3163	0.8045	0.5955	0.4947	0.5375	0.4470
Avg. Second Bid Price above Spot	2.6627	2.0447	2.1769	1.9259	1.8169	1.2807
Avg. First Ask Price below Spot	1.3674	0.8677	0.6561	0.5436	0.6224	0.5237
Avg. Second Ask Price below Spot	2.7299	1.9204	2.0791	1.8928	1.8011	1.3923
Avg. First Ask Price above Spot	1.3453	0.8262	0.6132	0.5112	0.5562	0.4626
Avg. Second Ask Price above Spot	2.7410	2.0988	2.2374	1.9677	1.8582	1.3072

TABLE 3

GS

	Q1	Q2	Q3	Q4	Q5	Q6
Spot Price	205.8890	178.8911	170.8944	161.5178	85.6764	86.4233
Avg. First Maturity	0.2669	0.2761	0.2656	0.2726	0.2620	0.2647
Avg. Second Maturity	1.3954	1.2039	1.1959	1.3659	1.4062	1.2889
Avg. First Interest Rate	0.0444	0.0297	0.0257	0.0279	0.0230	0.0065
Avg. Second Interest Rate	0.0392	0.0264	0.0294	0.0311	0.0227	0.0103
Avg. First Div. Yld.	0.0038	0.0053	0.0048	0.0058	0.0095	0.0234
Avg. Second Div. Yld.	0.0055	0.0070	0.0066	0.0080	0.0157	0.0143
Avg. First Strike below Spot	172.5547	152.8181	145.4529	137.6630	73.2500	73.1250
Avg. First Strike above Spot	235.2670	201.7718	192.0824	182.7132	98.5875	99.5625
Avg. Second Strike below Spot	173.7783	154.0553	145.5567	136.5889	73.2493	73.1250
Avg. Second Strike above Spot	237.1402	203.3560	193.9653	184.9455	98.3250	99.7303
Avg. First Bid Price below Spot	7.0581	6.6115	5.3806	7.7943	9.6847	8.2695
Avg. Second Bid Price below Spot	20.2126	17.6156	15.3161	19.3304	19.3282	18.9575
Avg. First Bid Price above Spot	8.4677	7.3763	6.2978	7.4759	8.5792	7.4613
Avg. Second Bid Price above Spot	29.4781	21.8284	20.7137	23.3501	20.5439	19.1864
Avg. First Ask Price below Spot	7.3142	6.8587	5.5272	8.2803	10.1389	8.4144
Avg. Second Ask Price below Spot	20.6261	16.9805	15.6486	20.3305	20.6422	19.3763
Avg. First Ask Price above Spot	8.7945	7.6288	6.4187	7.9397	9.1159	7.6175
Avg. Second Ask Price above Spot	30.8537	20.7746	21.0414	24.3448	22.1067	19.8048

TABLE 4

GS

	Q7	Q8	Q9	Q10	Q11	Q12
Spot Price	130.5452	157.5548	177.9900	161.1004	159.8382	142.6230
Avg. First Maturity	0.2758	0.2581	0.2596	0.2679	0.2629	0.2680
Avg. Second Maturity	1.3037	1.3271	1.5860	1.4137	1.2143	1.3958
Avg. First Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Strike below Spot	111.3750	133.9406	151.8660	136.6635	135.8750	121.2500
Avg. First Strike above Spot	150.8474	178.9397	200.1926	183.1701	180.9420	163.1896
Avg. Second Strike below Spot	111.5588	134.2188	152.1073	137.4234	136.3171	121.6864
Avg. Second Strike above Spot	151.1912	179.4285	199.9964	183.1231	180.9791	164.1250
Avg. First Bid Price below Spot	7.2675	4.8544	4.8628	3.7362	4.5174	4.1726
Avg. Second Bid Price below Spot	19.6627	17.0755	19.5505	14.1029	14.2102	15.6209
Avg. First Bid Price above Spot	6.2807	4.8359	5.3282	3.8135	4.2268	3.8194
Avg. Second Bid Price above Spot	19.4212	18.3623	22.4179	15.9371	15.0809	16.2977
Avg. First Ask Price below Spot	7.3844	4.9612	4.9821	3.8391	4.6495	4.2755
Avg. Second Ask Price below Spot	20.1608	17.6654	20.6238	14.6941	14.7540	16.0029
Avg. First Ask Price above Spot	6.4167	4.9531	5.4483	3.9244	4.3612	3.9217
Avg. Second Ask Price above Spot	20.0600	19.0731	23.6683	16.6605	15.7275	16.8088

TABLE 5

JPM

	Q1	Q2	Q3	Q4	Q5	Q6
Spot Price	39.3450	43.0062	40.4383	37.6808	35.4442	24.4726
Avg. First Maturity	0.2245	0.2808	0.2735	0.2333	0.2173	0.2744
Avg. Second Maturity	1.1767	1.2207	0.9771	0.7896	1.0275	1.2970
Avg. First Interest Rate	0.0424	0.0296	0.0256	0.0267	0.0236	0.0089
Avg. Second Interest Rate	0.0376	0.0264	0.0283	0.0287	0.0230	0.0132
Avg. First Div. Yld.	0.0257	0.0323	0.0332	0.0335	0.0381	0.0161
Avg. Second Div. Yld.	0.0308	0.0379	0.0368	0.0345	0.0365	0.0318
Avg. First Strike below Spot	33.6681	36.8206	34.5385	31.9375	30.2839	21.1644
Avg. First Strike above Spot	44.6517	49.0092	46.0548	42.8473	40.3281	27.8942
Avg. Second Strike below Spot	34.1567	37.0318	34.4160	31.7864	29.8635	20.5464
Avg. Second Strike above Spot	45.0655	49.6561	46.3749	42.6716	40.2559	27.7983
Avg. First Bid Price below Spot	1.1245	1.7480	1.4767	1.8811	3.0823	2.8912
Avg. Second Bid Price below Spot	3.5874	4.7679	3.5915	3.7742	6.2728	5.9152
Avg. First Bid Price above Spot	0.9930	1.4468	1.2797	1.6695	2.8129	2.6629
Avg. Second Bid Price above Spot	3.5251	4.0203	3.3235	3.7238	6.0316	5.9026
Avg. First Ask Price below Spot	1.2088	1.8578	1.5179	1.9742	3.1956	2.9541
Avg. Second Ask Price below Spot	3.9063	5.0783	3.6542	3.9758	6.5236	6.0870
Avg. First Ask Price above Spot	1.0925	1.5682	1.3223	1.7679	2.9392	2.7324
Avg. Second Ask Price above Spot	3.8789	4.2248	3.4066	3.9341	6.3487	6.1069

TABLE 6

JPM

	Q7	Q8	Q9	Q10	Q11	Q12
Spot Price	33.1520	38.6284	43.8825	41.0949	43.0198	38.4432
Avg. First Maturity	0.2787	0.2467	0.2111	0.2686	0.2837	0.2688
Avg. Second Maturity	1.0663	0.7896	0.8423	1.3049	1.0974	0.8361
Avg. First Interest Rate	0.0067	0.0037	0.0026	0.0026	0.0058	0.0034
Avg. Second Interest Rate	0.0106	0.0065	0.0051	0.0074	0.0090	0.0047
Avg. First Div. Yld.	0.0049	0.0043	0.0042	0.0039	0.0059	0.0051
Avg. Second Div. Yld.	0.0064	0.0051	0.0042	0.0050	0.0142	0.0115
Avg. First Strike below Spot	28.5431	33.3140	37.9094	36.3986	38.0559	34.4533
Avg. First Strike above Spot	37.1799	42.6482	47.9970	45.6780	47.4132	43.5051
Avg. Second Strike below Spot	27.7499	33.0409	37.9387	35.3541	37.2218	34.3735
Avg. Second Strike above Spot	38.0731	43.2298	49.1882	47.4179	49.3418	43.1352
Avg. First Bid Price below Spot	2.6247	1.5324	1.2899	1.2823	1.4769	1.5171
Avg. Second Bid Price below Spot	5.3603	3.3665	3.6014	4.0096	3.9125	3.7590
Avg. First Bid Price above Spot	2.6194	1.6819	1.4856	1.2039	1.3465	1.0833
Avg. Second Bid Price above Spot	5.8348	3.7621	3.7661	3.5283	3.2964	3.1273
Avg. First Ask Price below Spot	2.6769	1.5658	1.3195	1.3128	1.5146	1.5462
Avg. Second Ask Price below Spot	5.4862	3.4568	3.7079	4.1142	4.0110	3.8357
Avg. First Ask Price above Spot	2.6791	1.7190	1.5193	1.2362	1.3849	1.1114
Avg. Second Ask Price above Spot	6.0157	3.8846	3.8877	3.6364	3.3950	3.2009

TABLE 7

WFC

	Q1	Q2	Q3	Q4	Q5	Q6
Spot Price	30.9669	30.1230	27.4836	30.1961	29.8417	17.2202
Avg. First Maturity	0.2813	0.2815	0.2728	0.2591	0.2698	0.2728
Avg. Second Maturity	1.5430	1.3309	1.3191	1.3349	1.4117	1.3050
Avg. First Interest Rate	0.0467	0.0294	0.0270	0.0281	0.0227	0.0065
Avg. Second Interest Rate	0.0410	0.0263	0.0312	0.0313	0.0224	0.0104
Avg. First Div. Yld.	0.0324	0.0370	0.0393	0.0398	0.0390	0.0424
Avg. Second Div. Yld.	0.0381	0.0396	0.0394	0.0340	0.0315	0.0297
Avg. First Strike below Spot	26.1932	25.5341	23.3793	26.1248	26.0358	14.8932
Avg. First Strike above Spot	35.0676	34.4648	31.4400	34.1238	33.7505	19.7992
Avg. Second Strike below Spot	26.1674	25.5750	23.4179	25.7285	25.3033	14.6156
Avg. Second Strike above Spot	35.0604	34.2163	31.5230	34.4796	34.0296	19.7733
Avg. First Bid Price below Spot	0.9648	1.2410	0.9851	1.9339	3.2954	2.5609
Avg. Second Bid Price below Spot	2.9478	3.2936	2.6964	4.1903	6.2667	4.5430
Avg. First Bid Price above Spot	0.9238	1.1098	0.8582	1.6048	2.6488	2.3307
Avg. Second Bid Price above Spot	3.1993	3.1701	2.6653	3.9148	5.6953	4.4963
Avg. First Ask Price below Spot	1.0511	1.3426	1.0748	2.0921	3.5098	2.6759
Avg. Second Ask Price below Spot	3.3836	3.5628	2.8810	4.6264	6.7802	4.8182
Avg. First Ask Price above Spot	1.0159	1.2151	0.9481	1.7746	2.8941	2.4631
Avg. Second Ask Price above Spot	3.4910	3.5469	2.8894	4.4648	6.4190	4.8646

TABLE 8

WFC

	Q7	Q8	Q9	Q10	Q11	Q12
Spot Price	21.8491	25.8781	28.3260	27.5206	31.4716	26.8436
Avg. First Maturity	0.2653	0.2547	0.2681	0.2606	0.2594	0.2549
Avg. Second Maturity	1.2557	1.3277	1.4452	1.2980	1.1080	1.3159
Avg. First Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Interest Rate	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. Second Div. Yld.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Avg. First Strike below Spot	18.5042	22.1769	24.5985	24.1333	27.3234	23.3528
Avg. First Strike above Spot	24.7624	29.1128	31.9345	31.0555	34.7281	30.5766
Avg. Second Strike below Spot	18.6636	21.6832	23.7227	22.9853	26.5399	22.4307
Avg. Second Strike above Spot	25.4233	30.0096	32.4470	31.9857	35.5041	31.2107
Avg. First Bid Price below Spot	2.1514	1.3861	1.2813	0.9260	0.9732	0.9719
Avg. Second Bid Price below Spot	4.8352	4.2221	3.9607	2.6752	2.6019	3.2020
Avg. First Bid Price above Spot	2.1746	1.4094	1.1747	0.7788	0.8449	0.7620
Avg. Second Bid Price above Spot	4.7183	4.2972	4.3307	2.5122	2.5898	2.9836
Avg. First Ask Price below Spot	2.2443	1.4707	1.3441	0.9504	1.0050	0.9985
Avg. Second Ask Price below Spot	5.0596	4.4414	4.1381	2.7548	2.6912	3.3259
Avg. First Ask Price above Spot	2.2852	1.5008	1.2365	0.8048	0.8764	0.7888
Avg. Second Ask Price above Spot	5.0010	4.5963	4.5494	2.5897	2.6906	3.1049

TABLE 9

Calibration Results on BAC of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.4202	0.6275	-0.4038	0.4802	0.0099	0.0048	24.6438	2.8165
Std. Dev.	0.1228	0.2486	0.3277	0.0881	0.0071	0.0027	10.1547	2.4045
Percentage			Quantiles					
1	0.2242	0.1578	-1.4733	0.2064	0.0014	0.0003	7.9354	0.3378
5	0.2919	0.2638	-1.0663	0.2991	0.0023	0.0008	11.5932	0.4218
10	0.3020	0.3328	-0.8300	0.3572	0.0028	0.0014	13.6045	0.4857
25	0.3337	0.4332	-0.5623	0.4441	0.0044	0.0028	17.0002	1.1010
50	0.3979	0.5898	-0.3296	0.5017	0.0081	0.0048	23.1412	2.4978
75	0.4699	0.8225	-0.2025	0.5325	0.0132	0.0063	29.7766	3.6711
90	0.6700	0.9764	-0.0396	0.5660	0.0190	0.0084	40.6698	4.8467
95	0.7047	1.0339	0.0177	0.5763	0.0251	0.0104	44.9378	7.7727
99	0.7429	1.1989	0.0876	0.6594	0.0334	0.0121	50.1391	12.8423

Goodness of Fit Metrics

	rmse	aae	ape	number of options
Mean	0.0738	0.0562	0.0316	35.8523
Std. Dev.	0.0683	0.0481	0.0171	10.2221

TABLE 10

Calibration Results on GS of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3527	0.6832	-0.3524	0.4696	0.0056	0.0079	34.8995	4.0117
Std. Dev.	0.0388	0.2632	0.1493	0.0578	0.0058	0.0037	15.2737	3.2182
Percentage			Quantiles					
1	0.2736	0.2994	-0.7598	0.3448	0.0000	0.0011	16.7497	0.6302
5	0.3059	0.3876	-0.6798	0.3667	0.0001	0.0016	18.6538	0.6523
10	0.3153	0.4018	-0.5646	0.3826	0.0002	0.0020	19.8705	0.7349
25	0.3300	0.5329	-0.4183	0.4280	0.0006	0.0055	23.7206	2.0825
50	0.3439	0.6120	-0.3335	0.4832	0.0027	0.0080	31.9683	3.3678
75	0.3751	0.7972	-0.2680	0.5195	0.0097	0.0107	41.4322	4.9972
90	0.3865	1.0111	-0.2148	0.5307	0.0155	0.0125	53.3556	7.6455
95	0.4163	1.1154	-0.1022	0.5489	0.0163	0.0137	70.9174	11.0731
99	0.5001	1.8070	-0.0013	0.5680	0.0176	0.0145	84.5815	16.0059

	Goodness of Fit Metrics			
	rmse	aae	ape	number of options
Mean	0.5957	0.3662	0.0329	77.2869
Std. Dev.	0.9271	0.4595	0.0352	16.5025

TABLE 11

Calibration Results on JPM of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3647	0.6998	-0.4213	0.4797	0.0052	0.0102	19.5679	3.0458
Std. Dev.	0.0768	0.1536	0.1733	0.0507	0.0046	0.0055	9.1157	1.6106
Percentage			Quantiles					
1	0.2261	0.3615	-0.9429	0.3818	0.0000	0.0033	11.7712	0.7222
5	0.2658	0.4466	-0.7348	0.3914	0.0003	0.0044	12.3632	0.7436
10	0.2905	0.4705	-0.6885	0.4020	0.0003	0.0051	13.1412	0.9873
25	0.3198	0.5889	-0.5387	0.4426	0.0009	0.0065	14.4034	1.7811
50	0.3486	0.7141	-0.3557	0.4914	0.0038	0.0100	15.7181	2.8846
75	0.3977	0.8099	-0.2987	0.5198	0.0088	0.0109	19.5962	4.1092
90	0.4488	0.8939	-0.2573	0.5379	0.0125	0.0161	37.4757	4.8703
95	0.5592	0.9530	-0.2290	0.5537	0.0137	0.0208	44.1697	5.9696
99	0.6000	0.9931	-0.2013	0.5690	0.0164	0.0319	47.4503	7.8637

	Goodness of Fit Metrics			
	rmse	aae	ape	number of options
Mean	0.0948	0.0705	0.0290	58.0802
Std. Dev.	0.0496	0.0335	0.0128	16.0066

TABLE 12

Calibration Results on WFC of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3525	0.7594	-0.3678	0.4600	0.0104	0.0153	26.1688	2.6124
Std. Dev.	0.1044	0.2358	0.4022	0.0674	0.0071	0.0090	15.2591	1.3752
Percentage			Quantiles					
1	0.1583	0.2668	-1.0809	0.3429	0.0000	0.0041	11.2826	0.5197
5	0.1961	0.3156	-0.9423	0.3520	0.0013	0.0052	11.4094	0.5487
10	0.2161	0.3950	-0.8554	0.3609	0.0019	0.0069	11.8162	0.6205
25	0.3090	0.6292	-0.5525	0.4064	0.0052	0.0094	13.5901	1.5983
50	0.3442	0.7750	-0.2927	0.4576	0.0094	0.0119	21.3955	2.8544
75	0.3790	0.9263	-0.2671	0.5209	0.0134	0.0207	31.8577	3.6566
90	0.4731	1.0302	-0.1614	0.5462	0.0217	0.0267	51.3587	4.1181
95	0.6050	1.1378	0.3152	0.5535	0.0249	0.0319	58.2857	4.3507
99	0.6801	1.2210	1.2665	0.5575	0.0288	0.0478	65.9812	5.8069
			Goodness of Fit Metrics					
	rmse	aae	ape	number of options				
Mean	0.1002	0.0571	0.0349	43.7173				
Std. Dev.	0.0785	0.0543	0.0161	10.5690				

TABLE 13

BAC

Avg. Absolute Sensitivity of Bid Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	10.4710	1.0105	3.5254	3.0530	6.3415	23.0437	0.0022	0.4188
Q2	9.2505	1.9627	3.0452	2.9245	5.9694	20.8324	0.0120	0.8161
Q3	7.6859	0.3394	1.5040	2.4363	4.6130	16.2945	0.0059	0.1950
Q4	5.3147	8.1879	2.1513	3.1814	4.5034	13.8625	0.0259	2.3832
Q5	2.8542	6.1314	0.4683	1.7148	2.3209	6.4090	0.0146	1.9420
Q6	0.9038	2.0212	0.4779	1.0735	0.3482	0.8335	0.0073	0.2325
Q7	0.7655	0.0802	0.2469	0.6566	0.9132	2.4890	0.0111	0.3985
Q8	1.6000	1.3382	0.2238	0.9182	1.2838	3.9373	0.0027	0.2407
Q9	1.3999	3.7286	0.3688	0.8397	1.2418	4.0059	0.0007	0.1095
Q10	1.7163	0.1370	0.2809	0.5931	1.1011	3.8051	0.0001	0.0009
Q11	1.8246	0.1310	0.4382	0.6966	1.2759	4.3552	0.0009	0.0781
Q12	1.1575	0.0601	0.2315	0.4523	0.8399	2.9145	0.0013	0.0384

TABLE 14

BAC

Avg. Absolute Sensitivity of Ask Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	13.2010	1.2419	4.2619	3.8332	29.5953	8.5843	0.0026	0.4939
Q2	11.2021	2.3509	3.6084	3.5347	25.9518	7.5696	0.0139	0.9294
Q3	8.5567	0.3724	1.6361	2.6986	18.1903	5.2897	0.0065	0.2122
Q4	6.1925	9.0695	2.3755	3.6247	16.1250	5.2167	0.0301	2.6824
Q5	3.3463	7.1821	0.7001	2.0212	7.5857	2.7948	0.0183	2.2200
Q6	0.9065	2.1699	0.4628	1.0620	0.9233	0.3869	0.0079	0.2527
Q7	0.8236	0.0859	0.2591	0.7036	2.6916	0.9735	0.0117	0.4226
Q8	1.7102	1.4329	0.2362	0.9803	4.2734	1.3668	0.0028	0.2533
Q9	1.5193	4.0076	0.3944	0.9082	4.3566	1.3617	0.0008	0.1174
Q10	1.8419	0.1474	0.2961	0.6346	4.0666	1.2015	0.0001	0.0010
Q11	1.9448	0.1393	0.4604	0.7418	4.6454	1.3778	0.0009	0.0841
Q12	1.2233	0.0635	0.2412	0.4771	3.0870	0.8932	0.0014	0.0403

TABLE 15

GS

Avg. Absolute Sensitivity of Bid Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	311.2989	13.9577	107.7851	105.5504	190.0397	572.9021	0.3947	14.3101
Q2	186.6134	43.3509	76.5194	74.7206	137.4267	423.4965	0.2626	5.0556
Q3	205.6936	7.2845	56.8686	69.2453	130.4310	419.1693	0.2074	5.9646
Q4	142.6868	16.7028	62.7668	69.1106	114.3479	359.6128	0.1013	33.1307
Q5	29.3733	18.9277	19.6688	29.2601	43.5058	116.5837	0.1429	45.5001
Q6	30.8408	18.1780	21.9369	27.9824	46.9547	123.9158	0.1479	35.0905
Q7	94.1940	28.8316	53.8823	55.4381	93.5647	275.5709	0.1439	18.5643
Q8	135.6020	16.6353	45.3832	52.4444	99.2491	325.1180	0.0110	0.1649
Q9	231.0429	22.2158	44.0546	71.4349	128.6162	417.1284	0.0000	0.0002
Q10	211.6656	7.8598	29.9143	48.1331	107.5315	371.6249	0.0317	0.6146
Q11	156.8463	39.7682	32.3852	50.1983	97.1068	324.2034	0.0704	5.7242
Q12	143.4408	11.5995	28.8135	40.5014	87.7912	302.6896	0.0612	1.8374

TABLE 16

GS

Avg. Absolute Sensitivity of Ask Prices

	sigma	nu	theta	gamma	lamda	eta	c	a	a
Q1	370.9498	16.8426	137.2718	120.8807	693.6009	224.2542	0.7640	22.5801	0.4188
Q2	208.2717	50.4748	87.7557	83.2275	475.7938	152.8606	0.2810	5.5437	0.8161
Q3	218.1825	7.7445	59.4864	73.2449	449.9929	137.5364	0.2176	6.2750	0.1950
Q4	156.4715	19.4428	67.2470	75.9624	398.7843	126.0667	0.1078	41.6356	2.3832
Q5	34.7167	23.4577	23.1285	34.8659	142.0015	51.4699	0.1602	51.4357	1.9420
Q6	33.1332	19.3258	23.2737	29.9892	134.4594	49.8448	0.1561	37.0055	0.2325
Q7	101.3274	30.6987	57.3848	59.6618	303.6781	100.3334	0.1515	19.5147	0.3985
Q8	149.5967	18.2027	48.9103	57.6390	368.1363	109.4115	0.0116	0.1748	0.2407
Q9	270.8618	25.4064	50.0028	83.5783	490.2721	157.1312	0.0000	0.0002	0.1095
Q10	239.7168	8.8620	32.7483	54.4841	421.3487	126.5150	0.0355	0.6767	0.0009
Q11	173.8655	43.7590	34.9426	55.6975	364.3916	109.8604	0.0760	6.1350	0.0781
Q12	154.5127	12.6051	30.4498	43.5261	329.5058	95.6251	0.0660	1.9652	0.0384

TABLE 17

JPM

Avg. Absolute Sensitivity of Bid Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	11.3007	0.4486	3.5851	3.8444	7.0510	23.8413	0.0032	0.0496
Q2	11.2991	0.4915	3.7032	4.1838	7.4770	24.7995	0.0129	0.4582
Q3	10.6527	1.3533	3.2423	4.0221	7.2526	24.7319	0.0053	0.0535
Q4	9.0941	0.4238	3.3948	5.0402	6.8024	22.1556	0.0126	0.5170
Q5	5.4452	2.7624	3.1783	5.4121	6.4796	18.2728	0.0113	3.6528
Q6	2.8834	0.7233	0.9501	2.6134	3.4402	9.4151	0.0088	2.6238
Q7	4.9745	3.8017	2.1828	4.1575	5.7134	16.2723	0.0285	1.9407
Q8	4.7235	2.2117	3.9013	3.8218	5.6913	17.8972	0.0032	0.1520
Q9	9.6260	0.6360	3.2025	4.2087	6.7605	22.3807	0.0145	0.4422
Q10	10.5319	0.3464	3.1519	3.4584	6.8364	23.5496	0.0082	0.2431
Q11	10.9030	0.3816	3.2755	3.5070	7.0082	24.3745	0.0148	0.8580
Q12	8.3031	2.3362	3.3461	3.4727	6.5217	22.7208	0.0189	0.5873

TABLE 18

JPM

Avg. Absolute Sensitivity of Ask Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	14.4291	0.5643	4.2605	4.8444	31.5361	9.1930	0.0040	0.0610
Q2	13.4643	0.5786	4.1962	4.9338	30.0670	9.0951	0.0150	0.5221
Q3	11.5962	1.4882	3.4578	4.3569	27.0859	7.9886	0.0056	0.0562
Q4	10.2526	0.4831	3.6497	5.7012	25.1404	7.7996	0.0138	0.5753
Q5	6.2124	3.0679	3.5798	6.2021	21.2783	7.3612	0.0122	3.9446
Q6	3.1510	0.7767	1.0064	2.8346	10.3453	3.7045	0.0094	2.8029
Q7	5.3758	4.0420	2.3175	4.4782	17.8822	6.0990	0.0302	2.0528
Q8	5.1022	2.3476	4.1545	4.1063	19.8063	6.1142	0.0034	0.1617
Q9	10.4174	0.6763	3.4247	4.5487	24.8567	7.3429	0.0153	0.4661
Q10	11.4340	0.3767	3.3437	3.7461	26.2511	7.4109	0.0087	0.2569
Q11	11.6592	0.4035	3.4549	3.7456	26.3915	7.5790	0.0157	0.9024
Q12	8.8202	2.4653	3.5190	3.6819	24.4126	6.9912	0.0198	0.6139

TABLE 19

WFC

Avg. Absolute Sensitivity of Bid Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	5.3001	0.2725	1.9649	1.8887	3.3024	10.9900	0.0223	0.1185
Q2	5.8503	0.2676	1.6546	2.2449	3.6642	12.0948	0.0060	0.1027
Q3	5.3023	0.2329	1.0639	1.8413	3.0769	10.5091	0.0033	0.0326
Q4	3.6018	1.2451	2.9792	2.9738	4.0520	11.8650	0.0058	0.9846
Q5	4.8127	1.4187	1.9088	3.4256	4.9108	13.2858	0.0113	5.7315
Q6	1.4826	0.5563	0.3696	1.4117	1.7335	4.7806	0.0036	1.9424
Q7	1.3889	2.3437	1.5624	2.2234	2.8909	7.9215	0.0107	1.8542
Q8	1.5090	2.2215	2.1112	2.3592	3.1582	9.5688	0.0065	0.4859
Q9	2.0233	7.4582	2.0094	2.2794	3.5653	11.1895	0.0080	0.6362
Q10	4.4386	0.1681	1.3612	1.5981	3.0392	10.5060	0.0019	0.0655
Q11	4.7305	0.1546	1.8132	1.8323	3.3769	11.9699	0.0035	0.3940
Q12	3.6042	0.1358	0.9340	1.5337	2.6950	9.4775	0.0070	0.3198

TABLE 20

WFC

Avg. Absolute Sensitivity of Ask Prices

	sigma	nu	theta	gamma	lamda	eta	c	a
Q1	7.7040	0.3857	2.5648	2.6787	16.4817	4.9760	0.0289	0.1699
Q2	7.5394	0.3426	1.9876	2.8598	15.8624	4.8754	0.0073	0.1239
Q3	6.5407	0.2872	1.2436	2.2503	13.0797	3.9058	0.0040	0.0387
Q4	4.5244	1.7042	3.6668	3.8326	15.9519	5.1983	0.0070	1.1764
Q5	6.1362	1.7629	2.2734	4.3499	17.3585	5.9658	0.0131	6.6677
Q6	1.7648	0.6349	0.4164	1.6774	5.6579	2.0543	0.0042	2.2289
Q7	1.5919	2.6555	1.7638	2.5392	9.1802	3.3332	0.0117	2.0539
Q8	1.7442	2.5332	2.4199	2.7237	11.3405	3.7330	0.0073	0.5478
Q9	2.3102	8.3326	2.2614	2.5750	13.0196	4.0717	0.0089	0.7072
Q10	4.8179	0.1816	1.4456	1.7290	11.6368	3.2965	0.0020	0.0700
Q11	5.1648	0.1682	1.9322	1.9986	13.2143	3.7614	0.0037	0.4222
Q12	3.9403	0.1477	0.9957	1.6697	10.6150	2.9454	0.0077	0.3444

TABLE 21

Results of Regressing Capital on Calibrated Parameters

	BAC	GS	JPM	WFC
Constant	0.1987	-0.7087	0.6370	14.8208
t-stat	0.50	-2.97	2.44	6.98
sigma	8.6572	8.4179	3.4465	-0.9939
t-stat	13.88	23.36	11.44	-0.49
nu	-0.5490	-0.5233	0.2695	-7.9458
t-stat	-1.91	-6.91	1.49	-10.92
theta	-0.6968	-1.7883	-1.5137	7.0250
t-stat	-4.18	-18.02	-15.34	25.23
gamma	-6.4725	-4.9149	-4.6767	-10.4655
t-stat	-9.79	-13.88	-12.59	-3.26
lamda	341.0639	372.3114	364.5014	384.9392
t-stat	35.79	93.36	113.44	22.25
eta	267.0748	347.8530	334.7662	372.3261
t-stat	12.68	56.20	123.36	22.76
c	0.0360	0.0039	0.0107	0.0501
t-stat	6.67	3.51	5.94	4.14
a	-0.0518	-0.0352	-0.0582	-0.4749
t-stat	-1.95	-6.59	-5.51	-4.22
R2	94.08	99.26	99.76	95.77